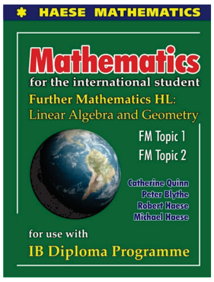


ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

FURTHER MATHEMATICS HL: Linear Algebra and Geometry

First edition - 2015 first reprint

The following erratum was made on 17/Jun/2022

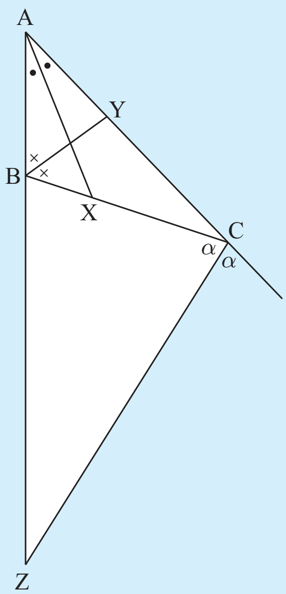
page 228 **SECTION 2Q** demo no longer supported, icon removed. (Adobe Flash Player)

The following erratum was made on 14/Oct/2019

pages 187 and 188 **SECTION 2K Example 20,** solution should read:

Example 20

In a triangle two angles are bisected internally, and the third angle is bisected externally. Prove that the points where the angle bisectors meet the triangle's sides are collinear.



Let the triangle be ABC, and the internal angle bisectors at A and B meet [BC] and [AC] at X and Y respectively. Let the external angle at C be bisected by [CZ] where Z lies on [AB] produced.

By the angle bisector theorem, as [AX] bisects $B\widehat{A}C$,

$$\frac{AB}{AC} = \frac{BX}{XC} \Rightarrow \frac{BX}{XC} = \frac{AB}{AC} \quad .... (1)$$

Likewise, as [BY] bisects $A\widehat{B}C$,

$$\frac{BA}{BC} = \frac{AY}{YC} \Rightarrow \frac{CY}{YA} = \frac{BC}{AB} \quad .... (2)$$

Also, as [CZ] bisects the external angle,

$$\frac{CA}{CB} = \frac{AZ}{BZ} \Rightarrow \frac{AZ}{ZB} = -\frac{AC}{BC} \quad .... (3)$$

From (1), (2), and (3), $\dfrac{BX}{XC}.\dfrac{CY}{YA}.\dfrac{AZ}{ZB} = \dfrac{AB}{AC}.\dfrac{BC}{AB}.\left(-\dfrac{AC}{BC}\right)$

$$= -1$$

$\Rightarrow$ X, Y, and Z are collinear. {converse of Menelaus' theorem}

The following erratum was made on 12/Sep/2017

page 272 **Worked Solutions EXERCISE 1K.1 Question 4 a,** should not exclude circles being considered as ellipses:

4 From **3**, $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{|\mathbf{A}|}\begin{pmatrix} dx' - by' \\ -cx' + ay' \end{pmatrix}$

$\therefore\ x^2 + y^2 = 1$ becomes

$$\left(\frac{dx' - by'}{|\mathbf{A}|}\right)^2 + \left(\frac{-cx' + ay'}{|\mathbf{A}|}\right)^2 = 1$$

$\therefore\ d^2x'^{\,2} - 2bdx'y' + b^2y'^{\,2} + c^2x'^{\,2} - 2acx'y' + a^2y'^{\,2}$
$= |\mathbf{A}|^2$

$\therefore\ (c^2 + d^2)x'^{\,2} - 2(ac + bd)x'y' + (a^2 + b^2)y^2 = |\mathbf{A}|^2$

a An ellipse has the form $Ax^2 + By^2 = C$
So, we require $-2(ac + bd) = 0$

b A circle has the form $Ax^2 + By^2 = C$ where $A = B$.
So, we require $-2(ac + bd) = 0$
and $(c^2 + d^2) = (a^2 + b^2)$

The following erratum was made on 31/Jul/2017

page 267 **Worked Solutions EXERCISE 1J.2 Question 4 c,** should read:

4 c $$\mathrm{T}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} y \\ -z \\ x + y \end{pmatrix}$$

$$= x\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + z\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\text{So,}\quad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

or $$\mathrm{T}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\quad \mathrm{T}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

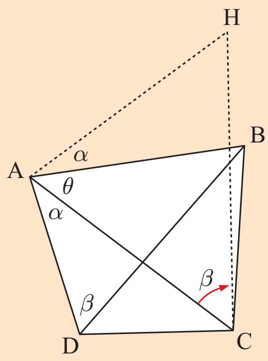
and $$\mathrm{T}\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\text{So,}\quad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

The following erratum was made on 28/Jul/2017

page 179 **SECTION 2J Proof for the converse for Ptolemy's theorem for cyclic quadrilaterals,** replace entirely with:

Proof:



Let $C\widehat{A}D = \alpha$, $A\widehat{D}B = \beta$, and $B\widehat{A}C = \theta$.

Choose a point H on the same side of AC as B, such that $B\widehat{A}H = \alpha$ and $A\widehat{C}H = \beta$.

Now $C\widehat{A}H = D\widehat{A}B = \alpha + \theta$

and $A\widehat{C}H = A\widehat{D}B = \beta$

$\therefore$ $\triangle$s ACH and ADB are equiangular and therefore similar.

$$\therefore \quad \underbrace{\frac{CH}{DB} = \frac{AC}{AD}}_{(1)} \; \underbrace{= \frac{AH}{AB}}_{(2)}$$

Rearranging (1), $CH = \dfrac{AC.DB}{AD}$ .... (3)

Given (2) and that $B\widehat{A}H = D\widehat{A}C = \alpha$, in $\triangle$s BAH and DAC we have two sides in the same ratio and the included angle between them equal.

$\therefore$ $\triangle$s BAH and DAC are similar.

$$\therefore \quad \frac{BH}{CD} = \frac{AB}{AD}$$

$\therefore$ $BH = \dfrac{AB.CD}{AD}$ .... (4)

$$\text{Using (3) and (4),} \quad CH - BH = \frac{AC.DB}{AD} - \frac{AB.CD}{AD}$$
$$= \frac{AC.BD - AB.CD}{AD}$$
$$= \frac{BC.AD}{AD} \quad \{\text{by the assumption of the converse}\}$$
$$= BC$$

$\therefore$ $BC + BH = CH$, which can only be true if B, C, and H are collinear.

$\therefore$ $A\widehat{C}B = \beta$

$\therefore$ $A\widehat{C}B = A\widehat{D}B$ are equal angles subtended by [AB].

$\therefore$ ABCD is a cyclic quadrilateral.

Note: this pushes questions 3 and 4 in Exercise 2J from page 180 to page 181.

The following errata were made on or before 09/Jan/2017

page 65 **SECTION 1I Subspaces,** third fact-box should read:

Every subspace of $\mathbb{R}^n$ contains the zero vector $\mathbf{0}$.

page 184 **SECTION 2K Example 17,** question should read:

Example 17

LMN is a triangle. X is on [LM], Y is on [MN], and Z is on [NL]. LX = 4 cm, MY = 3 cm, NZ = 5 cm, YN = 6 cm, ZL = 2 cm, and XM = 5 cm.

Prove that [MZ], [NX], and [LY] are concurrent.

The following errata were made on or before 06/Dec/2016

page 42 **EXERCISE 1F.1 Question 2 d,** should read:

2 Find the determinant of each of the following matrices. Hence find the inverse, if it exists.

a $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ **c** $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ **d** $\begin{pmatrix} 5 & 0 \\ 3 & 0 \end{pmatrix}$ **e** $\begin{pmatrix} a & a \\ -a & 1 \end{pmatrix}$

page 47 **EXERCISE 1F.3 Question 4,** should read:

4 For what values of a and b does $\begin{pmatrix} a^2 & 1 & 1 \\ 0 & a & b \\ 1 & 0 & 0 \end{pmatrix}$ have an inverse?

page 48 **EXERCISE 1F.4 Question 3,** should read:

3 If $\mathbf{A}$ is 3×3, explain why $|k\mathbf{A}| = k^3 |\mathbf{A}|$.

page 68 **EXAMPLE 30 Solution,** second to last line should read:

Since $|\mathbf{A}| = 0$, $\mathbf{A}^{-1}$ does not exist.

$\therefore$ $\mathbf{Ac} = \mathbf{x}$ does not have a unique solution $\mathbf{c} = \mathbf{A}^{-1}\mathbf{x}$ for each vector $\mathbf{x}$.

Thus $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ do not span $\mathbb{R}^3$.

page 100 **EXAMPLE 54 Answer part b,** last 3 lines should read:

$\Rightarrow \quad -x' + 3y' = 2x' + 2y' - 4$

$\Rightarrow \quad 3x' - y' = 4$

Hence $y = 2x - 1 \xrightarrow{T} 3x - y = 4$

page 112 **EXERCISE 1K.4 Question 8 c,** should read:

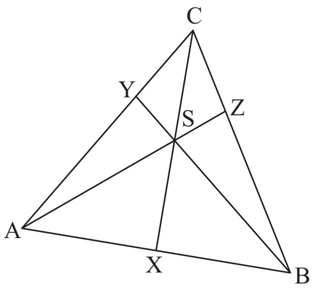
8 **a** Find the transformation matrix for a vertical stretch with scale factor k.

b Find the single transformation matrix for a vertical stretch with scale factor 2, followed by a reflection in the line $y = -2x$.

c What effect does the composition of transformations in **b** have on sense and area?

page 185 **EXERCISE 2K.1 Question 3**, should be replaced entirely with:

3



In the diagram, $BZ : ZC = 2 : 1$ and $AY : YC = 3 : 2$.

a Find the ratio in which X divides [AB].

b Find the ratio in which S divides [AZ].

page 242 **Worked Solutions EXERCISE 1F.1 Question 7 b**, should read:

7 **b** $\mathbf{A} = \begin{pmatrix} k-5 & -3 \\ 2 & k \end{pmatrix}$

$$\begin{aligned} \therefore\ \det \mathbf{A} &= (k-5)(k) - (-3)(2) \\ &= k^2 - 5k + 6 \\ &= (k-3)(k-2) \end{aligned}$$

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{(k-3)(k-2)} \begin{pmatrix} k & 3 \\ -2 & k-5 \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{k}{(k-3)(k-2)} & \dfrac{3}{(k-3)(k-2)} \\ \dfrac{-2}{(k-3)(k-2)} & \dfrac{k-5}{(k-3)(k-2)} \end{pmatrix} \end{aligned}$$

$\therefore$ $\mathbf{A}^{-1}$ exists provided $(k-3)(k-2) \neq 0$

$\therefore$ $k \neq 3$ or 2

page 273 **Worked Solutions EXERCISE 1K.2 Question 2 c**, should read:

2 **c** $\mathbf{A} = \begin{pmatrix} -\frac{5}{13} & \frac{12}{13} \\ -\frac{12}{13} & -\frac{5}{13} \end{pmatrix}$ where $|\mathbf{A}| = 1$

Since $\mathbf{A}$ has the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, $\mathbf{A}$ is a rotation matrix.

If the angle of rotation is θ, $\cos\theta = -\frac{5}{13}$ and $\sin\theta = -\frac{12}{13}$.

$\therefore$ $\tan\theta = \frac{12}{5}$ and $-\pi < \theta < -\frac{\pi}{2}$

$\therefore$ $\theta = \arctan\left(\frac{12}{5}\right)$

$\therefore$ the transformation is a clockwise rotation about O through $\pi - \arctan\left(\frac{12}{5}\right)$ (since $\cos\theta, \sin\theta < 0$).

page 274 **Worked Solutions EXERCISE 1K.2 Question 7 a**, should read:

7 $m = \tan\alpha = -2$

Now $\cos 2\alpha = \dfrac{1-m^2}{1+m^2}$ and $\sin 2\alpha = \dfrac{2m}{1+m^2}$

$\therefore$ $\cos 2\alpha = -\frac{3}{5}$ and $\sin 2\alpha = -\frac{4}{5}$

S has matrix $\mathbf{A} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$

a S has equations $x' = \dfrac{-3x - 4y}{5}$

$y' = \dfrac{-4x + 3y}{5}$

Thus $(-4, 2) \xrightarrow{S} \left(\dfrac{-3(-4) - 4(2)}{5}, \dfrac{-4(-4) + 3(2)}{5}\right)$

So, $(-4, 2) \xrightarrow{S} \left(\frac{4}{5}, \frac{22}{5}\right)$

pages 277 and 278 **Worked Solutions EXERCISE 1K.4 Question 2 b,** should be broken into two parts and read:

2 **a** T_1 is a reflection in $y = x$.

T_2 is a rotation about O through $\frac{\pi}{3}$.

T_1 has matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

T_2 has matrix $\mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$.

$\{\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\}$

b **i** $T_1 \circ T_2$ has matrix **AB** where

$$\mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

and $|\,\mathbf{AB}\,| = |\,\mathbf{A}\,|\,|\,\mathbf{B}\,| = -1 \times 1 = -1$

and **AB** has form $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$.

$\therefore$ $T_1 \circ T_2$ is a reflection in $y = (\tan\alpha)x$

where $\cos 2\alpha = \frac{\sqrt{3}}{2}$, $\sin 2\alpha = \frac{1}{2}$

$\therefore$ $2\alpha = \frac{\pi}{6}$

$\therefore$ $\alpha = \frac{\pi}{12}$

$\therefore$ $T_1 \circ T_2$ is a reflection in $y = (\tan\frac{\pi}{12})x$.

ii $T_2 \circ T_1$ has matrix **BA** where

$$\mathbf{BA} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

and $|\,\mathbf{BA}\,| = |\,\mathbf{B}\,|\,|\,\mathbf{A}\,| = 1 \times -1 = -1$

and **BA** has form $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$.

$\therefore$ $T_2 \circ T_1$ is a reflection in $y = (\tan\alpha)x$

where $\cos 2\alpha = -\frac{\sqrt{3}}{2}$, $\sin 2\alpha = \frac{1}{2}$

$\therefore$ $2\alpha = \frac{5\pi}{6}$

$\therefore$ $\alpha = \frac{5\pi}{12}$

$\therefore$ $T_2 \circ T_1$ is a reflection in $y = (\tan\frac{5\pi}{12})x$.

c As $T_1 \circ T_2$ and $T_2 \circ T_1$ represent different transformations,
$T_1 \circ T_2 \neq T_2 \circ T_1$.

page 278 **Worked Solutions EXERCISE 1K.4 Questions 3 and 4,** should read:

3 **a** T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$.

$\therefore$ $T_B \circ T_A$ has matrix **BA** where

$$\mathbf{BA} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos\phi\cos\theta - \sin\phi\sin\theta & -\sin\theta\cos\phi - \cos\theta\sin\phi \\ \sin\phi\cos\theta + \cos\phi\sin\theta & -\sin\phi\sin\theta + \cos\phi\cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{pmatrix}$$

which is the matrix for a rotation about O through $(\theta + \phi)$.

b T_A is a reflection in $y = (\tan\alpha)x$ and
T_B is a reflection in $y = (\tan\beta)x$.

T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{pmatrix}$.

Now $T_B \circ T_A$ has matrix **BA** where

BA
$$= \begin{pmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{pmatrix}\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos 2\beta\cos 2\alpha + \sin 2\beta\sin 2\alpha & \cos 2\beta\sin 2\alpha - \sin 2\beta\cos 2\alpha \\ \sin 2\beta\cos 2\alpha - \cos 2\beta\sin 2\alpha & \sin 2\beta\sin 2\alpha + \cos 2\beta\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos(2\beta-2\alpha) & -\sin(2\beta-2\alpha) \\ \sin(2\beta-2\alpha) & \cos(2\beta-2\alpha) \end{pmatrix}$$

which is the matrix for a rotation about O through an angle of $2(\beta - \alpha)$.

$\therefore$ $T_B \circ T_A$ is a rotation about O through $2(\beta - \alpha)$.

4 Let T_A be a reflection in $y = (\tan\alpha)x$, and
let T_B be a rotation about O through θ.

T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

Now $T_B \circ T_A$ has matrix **BA** where

BA
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta\cos 2\alpha - \sin\theta\sin 2\alpha & \cos\theta\sin 2\alpha + \sin\theta\cos 2\alpha \\ \sin\theta\cos 2\alpha + \cos\theta\sin 2\alpha & \sin\theta\sin 2\alpha - \cos\theta\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta+2\alpha) & \sin(\theta+2\alpha) \\ \sin(\theta+2\alpha) & -\cos(\theta+2\alpha) \end{pmatrix}$$

which is the matrix for a reflection in the line
$y = \left[\tan\left(\frac{\theta}{2} + \alpha\right)x\right]$

$\therefore$ $T_B \circ T_A$ is a reflection in $y = \left[\tan\left(\frac{\theta}{2} + \alpha\right)x\right]$.

page 278 **Worked Solutions EXERCISE 1K.4 Question 6,** first 5 lines should read:

6 T_A is unknown with matrix **A**.
T_B is a clockwise rotation about O through $\frac{\pi}{2}$.
$T_B \circ T_A$ is a reflection in $y = \frac{1}{2}x$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

For $T_B \circ T_A$, $\tan\alpha = \frac{1}{2}$

page 279 **Worked Solutions EXERCISE 1K.4 Questions 7 and 8,** should read:

7 $\mathrm{T_A}$ is a reflection in $y=-x$.

$\mathrm{T_B}$ is a rotation of $\frac{3\pi}{4}$ about O.

$\mathrm{T_A}$ has matrix $\mathbf{A}=\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

$\mathrm{T_B}$ has matrix $\mathbf{B}=\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$.

$\{\cos\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}},\quad \sin\left(\frac{3\pi}{4}\right)=\frac{1}{\sqrt{2}}\}$

$\mathrm{T_B}\circ\mathrm{T_A}$ has matrix **BA** where

$$\mathbf{BA}=\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}=\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathrm{T_B}\circ\mathrm{T_A}$ has equations $x'=\dfrac{x+y}{\sqrt{2}}$

$y'=\dfrac{x-y}{\sqrt{2}}$

and $(\mathbf{BA})^{-1}=\mathbf{BA}$ {as **BA** is a reflection}

$\therefore\ x=\dfrac{x'+y'}{\sqrt{2}},\quad y=\dfrac{x'-y'}{\sqrt{2}}$

$$\therefore\ y=2x-1 \xrightarrow{\mathrm{T_B}\circ\mathrm{T_A}} \frac{x'-y'}{\sqrt{2}}=2\left(\frac{x'+y'}{\sqrt{2}}\right)-1$$

$$\Rightarrow\ x'-y'=2x'+2y'-\sqrt{2}$$

$$\Rightarrow\ x'+3y'=\sqrt{2}$$

$$\therefore\ y=2x-1 \xrightarrow{\mathrm{T_B}\circ\mathrm{T_A}} x+3y=\sqrt{2}$$

8 **a** $\mathrm{A}=\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

b $\mathrm{T_A}$ is a vertical stretch of factor 2.

$\mathrm{T_B}$ is a reflection in $y=-2x$.

$\mathrm{T_A}$ has matrix $\mathbf{A}=\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

$\mathrm{T_B}$ has matrix $\mathbf{B}=\begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$.

$\{\tan\alpha=-2\quad\therefore\ \cos 2\alpha=\dfrac{1-4}{1+4}=-\frac{3}{5},$

$\sin 2\alpha=\dfrac{-4}{1+4}=-\frac{4}{5}\}$

$$\mathrm{T_B}\circ\mathrm{T_A}\ \text{has matrix}\ \mathbf{BA}=\begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}=\begin{pmatrix} -\frac{3}{5} & -\frac{8}{5} \\ -\frac{4}{5} & \frac{6}{5} \end{pmatrix}$$

c $\mathrm{T_B}\circ\mathrm{T_A}$ has $|\mathbf{BA}|=|\mathbf{B}|\,|\mathbf{A}|$

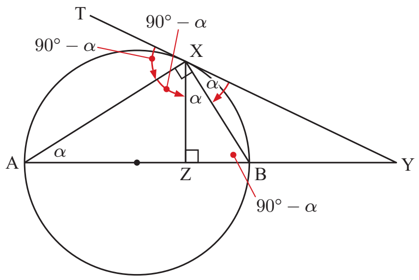
$=-1\times 2$

$=-2$

$\therefore\ \mathrm{T_B}\circ\mathrm{T_A}$ reverses sense and doubles area.

page 293 **Worked Solutions EXERCISE 2D Question 20**, should correctly calculate $X\widehat{A}Z$:

20



We join [XA] and [XB].

Let $Z\widehat{X}B = \alpha$

$A\widehat{X}B = 90°$ {angle in a semi-circle}

$\therefore\ Z\widehat{X}A = 90° - \alpha$

$\therefore\ X\widehat{A}Z = 180° - 90° - (90° - \alpha) = \alpha$

{angles in a triangle}

$\therefore\ B\widehat{X}Y = \alpha$ {angle between tangent and chord}

$Z\widehat{B}X = 180° - 90° - \alpha = 90° - \alpha$ {angles in a triangle}

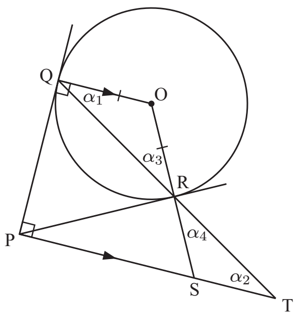
$\therefore\ A\widehat{X}T = 90° - \alpha$ {angle between tangent and chord}

$\therefore\ Z\widehat{X}B = B\widehat{X}Y = \alpha$ and $Z\widehat{X}A = A\widehat{X}T = 90° - \alpha$

$\therefore$ [XB] bisects $Z\widehat{X}Y$ and [XA] bisects $Z\widehat{X}T$.

page 293 **Worked Solutions EXERCISE 2D Question 24**, should read:

24



We join [OQ]. $O\widehat{Q}P = 90°$ {radius-tangent}

pages 308 and 309 **Worked Solutions EXERCISE 2K.1 Question 3**, should be replaced entirely with:

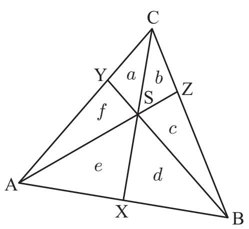
3 **a** By Ceva's theorem, $\dfrac{AX}{XB} \times \dfrac{BZ}{ZC} \times \dfrac{CY}{YA} = 1$

$\therefore\ \dfrac{AX}{XB} \times 2 \times \dfrac{2}{3} = 1$

$\therefore\ \dfrac{AX}{XB} = \dfrac{3}{4}$

$\therefore$ X divides [AB] in the ratio 3 : 4.

b



Now $c = 2b$

$f = \frac{3}{2}a$

$d = \frac{4}{3}e$

{areas of triangles are proportional to their bases if altitudes are equal}

Similarly, $e + d + c = 2(f + a + b)$

$\therefore\ e + \frac{4}{3}e + \cancel{2b} = 3a + 2a + \cancel{2b}$

$\therefore\ a = \frac{7}{15}e$ and so $f = \frac{7}{10}e$

Also, $4(a + f + e) = 3(b + c + d)$

$\therefore\ 4(\frac{7}{15}e + \frac{7}{10}e + e) = 3(b + 2b + \frac{4}{3}e)$

$\therefore\ \frac{26}{3}e = 4e + 9b$

$\therefore\ b = \frac{17}{21}e$

$\therefore$ AS : SZ $= f + a : b$

$= \frac{7}{15}\cancel{e} + \frac{7}{10}\cancel{e} : \frac{17}{21}\cancel{e}$

$= 49 : 34$