



MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) third edition - WORKED SOLUTIONS

Third edition - 2014 first reprint

The following erratum was made on 05/May/2020

page 918 EXERCISE 27A question 79, answer should read:

79
$$3 \sec 2x = \cot 2x + 3 \tan 2x, \quad -\pi \leqslant x \leqslant \pi$$

$$\therefore \frac{3}{\cos 2x} = \frac{\cos 2x}{\sin 2x} + 3 \frac{\sin 2x}{\cos 2x}, \quad -2\pi \leqslant 2x \leqslant 2\pi, \quad \cos 2x, \sin 2x \neq 0$$

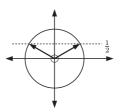
Multiplying all terms by $\sin 2x \cos 2x$ gives:

$$3\sin 2x = \cos^2 2x + 3\sin^2 2x$$

$$\therefore 3\sin 2x = 1 - \sin^2 2x + 3\sin^2 2x$$

$$\therefore 2\sin^2 2x - 3\sin 2x + 1 = 0$$

$$\therefore (2\sin 2x - 1)(\sin 2x - 1) = 0$$



The following errata were made on 29/Apr/2020

page 603 **EXERCISE 19A** question **8 c**, seventh line should give correct reason why x=-2:

8 **b**
$$f(x) = \frac{x}{\sqrt{2-x}}$$

Now $f(x)$ is a quotient where
$$u = x \text{ and } v = (2-x)^{\frac{1}{2}}$$

$$\therefore u' = 1 \text{ and } v' = \frac{1}{2}(2-x)^{-\frac{1}{2}}(-1)$$

$$\text{Now } f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{1(2-x)^{\frac{1}{2}} - x^{\frac{1}{2}}(2-x)^{-\frac{1}{2}}(-1)}{(2-x)^{\frac{2}{2}}}$$
c $f(x) = \frac{x}{\sqrt{2-x}} = -1$

$$\therefore x = -\sqrt{2-x}$$

$$\therefore x^2 + x - 2 = 0$$

$$\therefore (x+2)(x-1) = 0$$

$$\therefore x = -2 \text{ or } 1$$
but $f(1) = \frac{1}{\sqrt{2-1}} = 1$

$$\therefore x = -2$$

$$\therefore x = -2$$

$$\therefore the point of contact is $(-2, -1)$.$$

page 659 EXERCISE 20B question 18 b, last line of each part should have correct units:

18 b i When
$$t=0$$
,
$$\sin t = 0 \text{ and } \cos t = 1$$

$$\therefore \frac{dx}{dt} = 0 + 0$$

$$= 0 \text{ m per radian}$$
 iii When $t = \frac{\pi}{2}$,
$$\sin t = 1 \text{ and } \cos t = 0$$

$$\therefore \frac{dx}{dt} = 0 + 0$$

$$= 1 \text{ m per radian}$$

$$\therefore \frac{dx}{dt} = 0 + \sin(\frac{\pi}{2})$$

$$= 1 \text{ m per radian}$$

$$\approx 1.11 \text{ m per radian}$$

page 689 REVIEW SET 20C question 10 c, sign diagram should read:

10 c Now
$$a+r\neq 0$$
 so $\frac{dT}{d\theta}=0$ when $\sin\alpha-\frac{rv}{(a+r)w}=0$ $\therefore \sin\alpha=\frac{rv}{(a+r)w}$ Sign diagram for $\frac{dT}{d\theta}$ is:
$$\therefore T \text{ is minimised when } \sin\alpha=\frac{rv}{(a+r)w}$$

The following erratum was made on 21/Feb/2020

page 918 EXERCISE 27A question 79, last three lines and diagram should read:

79
$$3 \sec 2x = \cot 2x + 3 \tan 2x, \quad -\pi \leqslant x \leqslant \pi$$

$$\therefore \frac{3}{\cos 2x} = \frac{\cos 2x}{\sin 2x} + 3 \frac{\sin 2x}{\cos 2x}, \quad -2\pi \leqslant 2x \leqslant 2\pi$$

Multiplying all terms by $\sin 2x \cos 2x$ gives:

$$3\sin 2x = \cos^2 2x + 3\sin^2 2x$$

$$3\sin 2x = 1 - \sin^2 2x + 3\sin^2 2x$$

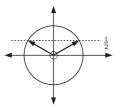
$$2\sin^2 2x - 3\sin 2x + 1 = 0$$

$$(2\sin 2x - 1)(\sin 2x - 1) = 0$$

$$\therefore \sin 2x = \frac{1}{2} \quad \{ \sin 2x \neq 1 \text{ otherwise } \cos 2x = 0 \}$$

$$\therefore 2x = \frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \text{ or } \frac{5\pi}{6}$$

$$\therefore x = \frac{-11\pi}{12}, \frac{-7\pi}{12}, \frac{\pi}{12}, \text{ or } \frac{5\pi}{12}$$



The following erratum was made on 20/Dec/2019

page 264 REVIEW SET 8C question 5, second line should read:

5 In the expansion of
$$\left(2x^2 - \frac{1}{x}\right)^6$$
, $a = 2x^2$, $b = -\frac{1}{x}$, $n = 6$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

For the constant term we let 12 - 3r = 0

The following errata were made on 15/May/2018

page 935 EXERCISE 27A question 123 a, should include sign diagrams:

123 a
$$f'(x)$$
 near a has sign diagram \xrightarrow{a}

$$f'(x)$$
 near b has sign diagram $+$ $+$ $+$ x , but $f'(b) \neq 0$

$$f'(x)$$
 near c has sign diagram $+$ $+$ $+$ x , but $f'(c) = 0$

page 937 **EXERCISE 27A** question **125 a**, should read:

125 a At any point
$$A(x, y)$$
, $\frac{dy}{dx} = \frac{y - 0}{x - \left(x - \frac{1}{2}\right)} = 2y$

$$\therefore \quad \frac{dx}{dy} = \frac{1}{2y}$$

The following erratum was made on 25/Jul/2017

page 631 EXERCISE 19D.1 question 12 a ii, should read:

12
$$f(t) = Ate^{-bt}, t \ge 0, A, b > 0$$

a ii
$$f''(t)=-Abe^{-bt}-(Abe^{-bt}-Ab^2te^{-bt}) \qquad \{\text{product rule}\}$$

$$=-2Abe^{-bt}+Ab^2te^{-bt}$$

$$=Abe^{-bt}(bt-2)$$

When
$$f''(t) = 0$$
 then $Abe^{-bt}(bt - 2) = 0$ but $A, b > 0$ and $e^{-bt} > 0$
 $\therefore bt - 2 = 0$ Sign diagram of $f''(t)$ is:

$$\therefore bt - 2 = 0$$

$$\therefore bt = 2$$

$$t = \frac{2}{b}$$

$$\therefore$$
 $t = \frac{2}{b}$ is a non-stationary point of inflection.

The following erratum was made on 13/Jun/2017

page 773 EXERCISE 22E.1 question 10 f, diagram should have correct function equation:

10 f $y = \tan\left(\frac{x}{2}\right)$

The following errata were made on 19/May/2017

pages 420 and 421 EXERCISE 14K.1 question 7, should read:

7
$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 + c_2 & b_3 + c_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 + c_1 & b_3 + c_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 + c_1 & b_2 + c_2 \end{vmatrix} \mathbf{k}$$

$$= (a_2(b_3 + c_3) - a_3(b_2 + c_2))\mathbf{i} - (a_1(b_3 + c_3) - a_3(b_1 + c_1))\mathbf{j}$$

$$+ (a_1(b_2 + c_2) - a_2(b_1 + c_1))\mathbf{k}$$

$$= (a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2)\mathbf{i} - (a_1b_3 + a_1c_3 - a_3b_1 - a_3c_1)\mathbf{j}$$

$$+ (a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1)\mathbf{k}$$

$$\mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\therefore \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k} + \begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ c_1 & c_2 \end{vmatrix} \mathbf{k}$$

$$= (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

$$+ (a_2c_3 - a_3c_2)\mathbf{i} - (a_1c_3 - a_3c_1)\mathbf{j} + (a_1c_2 - a_2c_1)\mathbf{k}$$

$$= (a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2)\mathbf{i} - (a_1b_3 + a_1c_3 - a_3b_1 - a_3c_1)\mathbf{j}$$

$$+ (a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1)\mathbf{k}$$

$$= \mathbf{a} \times (\mathbf{b} + \mathbf{c})$$

page 424 EXERCISE 14K.2 question 2 c, should read:

2
$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

$$= \pm \sqrt{1 - \left(\frac{1}{\sqrt{28}}\right)^2}$$

$$= \pm \sqrt{\frac{27}{28}}$$

But since θ is the angle between two vectors,

$$0^{\circ} \leqslant \theta \leqslant 180^{\circ}.$$

$$\therefore \sin \theta \geqslant 0$$

$$\therefore \sin \theta = \sqrt{\frac{27}{28}}$$

The following erratum was made on 22/Mar/2017

page 422 EXERCISE 14K.1 question 11, last line should read:

11
$$= \pm \frac{\sqrt{10}}{6} \begin{pmatrix} 4 \\ -5 \\ -7 \end{pmatrix}$$

The following erratum was made on 24/May/2016

page 506 EXERCISE 16B.2 question 4 d, diagram should have correct label:

4 a $z \mapsto z^*$. Reflection in the real axis.

b $z \mapsto -z$. Rotation of π about O.

 $z \mapsto -z^*$. Reflection in the imaginary axis.

d $z\mapsto -iz$. Clockwise rotation of $\frac{\pi}{2}$ about O.

