

ERRATA

Mathematics: Analysis and Approaches HL

First edition - 2024 fifth reprint

The following erratum was made on 21/Nov/2025

page 656 **CHAPTER 24 SECTION A**, second paragraph, should read:

However, if we **only consider the terms up to x^n** of the Maclaurin series expansion, we are left with an n th order polynomial approximation for the function.

The following erratum was made on 12/Sep/2025

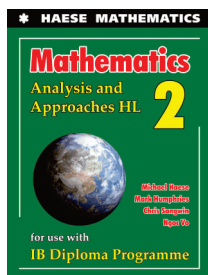
page 888 **ANSWERS REVIEW SET 22B**, question **17a**, should read:

17 a $(6 \ln 3 - 4) \text{ units}^2$ **b** $\left(3\sqrt[3]{4} - \frac{3}{4}\right) \text{ units}^2$

The following erratum was made on 24/Jul/2025

page 871 **ANSWERS EXERCISE 18H**, question **3**, replace with:

- 3** Repeated use of l'Hôpital's rule does not help as the expression in the limit becomes more and more complicated.



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Mathematics: Analysis and Approaches HL

First edition - 2023 fourth reprint

The following errata were made on 05/Sep/2024

page 437 CHAPTER 17 INVESTIGATION 1, question 1, should read:

What to do:

1 Use the binomial expansion $(x + h)^n = \sum_{r=0}^n \binom{n}{r} x^{n-r} h^r$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \dots + \binom{n}{n} h^n$$

and the first principles formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = x^n$ for $n \in \mathbb{N}$.

page 654 CHAPTER 24 OPENING PROBLEM, question e, should read:

e Can we write an *exact* expression for e^x in the form $e^x = \sum_{k=0}^{\infty} c_k x^k$ where $c_k \in \mathbb{R}$?

page 654 CHAPTER 24 HISTORICAL NOTE, third paragraph, should read:

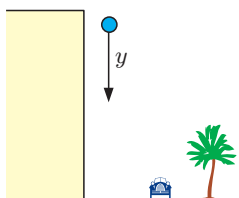
In the 14th century, the Indian mathematician **Mādhava of Sangamagrāma** discovered infinite series expansions for a number of trigonometric functions, as well as infinite series for

calculating π such as $\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$.

page 674 CHAPTER 24 SECTION A, example diagram for Object on a spring, should read:

For example:

A falling object



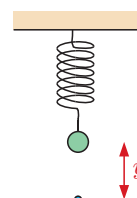
$$\frac{d^2 y}{dt^2} = 9.8$$

A parachutist



$$m \frac{dv}{dt} = mg - av^2$$

Object on a spring



$$m \frac{d^2 y}{dt^2} = -ky$$

page 700 CHAPTER 25 ACTIVITY 1, replace paragraphs 4 and 5 with:

In the case of a mass on a spring, k is determined by the mass of the object, and the stiffness of the spring.

In the case of a pendulum, k is determined by the weight on its end, and the length of the pendulum.

- The **standard deviation** of X is $\sigma = \sqrt{\text{Var}(\mathbf{X})} = \sqrt{np(1-p)}$.

The following erratum was made on 04/Jul/2024

page 250 CHAPTER 10 ACTIVITY, 7th line, should read:

Infinite descent was a historical precursor to mathematical induction. However, it is in fact a special form of proof by **contradiction**.

The following errata were made on 13/Mar/2024

page 133 EXERCISE 5I, question 9, should read:

- 9 $f(x)$ is a polynomial of degree n with leading coefficient 1. The sum of its **zeros** is 5, and the product of its **zeros** is -3 . Suppose $g(x) = [f(x)]^2$. Find:
- | | |
|---|---|
| a the degree of $g(x)$ | b the sum of the zeros of $g(x)$ |
| c the product of the zeros of $g(x)$ | d the y -intercept of $y = g(x)$. |

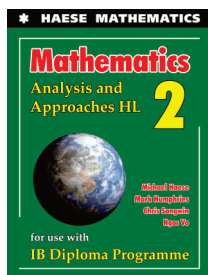
page 216 EXERCISE 9B, question 8, should read:

- 8 Consider a 3-digit number “ abc ”, $a \neq c$. Written backwards, it is “ cba ”. Let S be the **3-digit** result when the smaller of the two numbers is subtracted from the larger. When S is written backwards and the result is added to S , prove that the sum is always 1089.
- For example: 276 backwards is 672, so $S = 396$ and $396 + 693 = 1089$.

The following erratum was made on 23/Oct/2023

page 530 EXERCISE 19B, question 20, should read:

- 20 A cone has radius r cm and **fixed** slant height s cm.
- Find the ratio of $s : r$ which maximises the volume of a cone.



ERRATA

Mathematics: Analysis and Approaches HL

First edition - 2021 third reprint

The following erratum was made on 17/Apr/2023

page 821 **ANSWERS EXERCISE 5M**, question 4 f, should read:

- 4 a $-2.09 < x < 0.572$ or $x > 2.51$
 b $x < -1.66$ or $0.327 < x < 1.84$
 c $-2.55 < x < 0.290$ or $x \geq 2.26$ d $x \leq \frac{3}{2}$
 e $-1.20 < x < -0.125$ or $x > 1.33$ f $x < 0.924$

The following erratum was made on 15/Mar/2023

page 433 **REVIEW SET 16A**, question 9, should read:

- 9 Let $f(x) = \begin{cases} 11 - 2x, & x \geq 3 \\ x + 2, & x < 3 \end{cases}$.
 a Sketch the function $y = f(x)$. b Is f differentiable at $x = 3$?

The following errata were made on 06/Dec/2022

page 116 **EXERCISE 5C**, question 8 b, should read:

- 8 The graph of the polynomial $P(x)$ has three x -intercepts, α , β , and γ .
 a State the values of $P(\alpha)$, $P(\beta)$, and $P(\gamma)$.
 b Explain why $P(x)$ cannot have a **real linear** factor $(x - a)$ such that $a \neq \alpha, \beta$, or γ .

page 817 **ANSWERS EXERCISE 5C**, question 8 b, should read:

- 8 a $P(\alpha) = 0$, $P(\beta) = 0$, $P(\gamma) = 0$
 b If $P(x)$ has a **real linear** factor of $(x - a)$, then $P(a) = 0$ which implies that $P(x)$ has x -intercept a which is distinct from the x -intercepts of α , β , and γ .
 But $P(x)$ has only three x -intercepts and since $a \neq \alpha, \beta$, or γ , a cannot be one of them.
 $\therefore P(x)$ cannot have a factor of $(x - a)$.

The following erratum was made on 10/May/2022

page 386 CHAPTER 14 HISTORICAL NOTE, second paragraph should read:

HISTORICAL NOTE

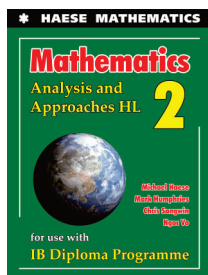
EULER'S BEAUTIFUL EQUATION

One of the most remarkable results in mathematics is known as **Euler's beautiful equation** $e^{i\pi} = -1$ named after **Leonhard Euler**.

It is called beautiful because it links together three great constants of mathematics: Euler's **number** e , the imaginary number i , and the ratio of a circle's circumference to its diameter π .

Harvard lecturer **Benjamin Pierce** said of $e^{i\pi} = -1$,

"Gentlemen, that is surely true, it is absolutely paradoxical; we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth."



ERRATA

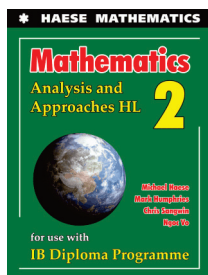
Mathematics: Analysis and Approaches HL

First edition - 2020 second reprint

The following erratum was made on 12/Nov/2020

page 667 **CHAPTER 24 EXAMPLE 6**, solution to part **c** should read:

$$\begin{aligned}
 \text{c} \quad & \arccos x \approx \frac{\pi}{2} - x - \frac{1}{6}x^3 \quad \{\text{using the first 3 terms}\} \\
 \therefore & \arccos(x^2) \approx \frac{\pi}{2} - x^2 - \frac{1}{6}x^6 \quad \text{for } |x| < 1 \\
 \therefore & \int_0^{0.4} \arccos(x^2) \, dx \approx \int_0^{0.4} \left(\frac{\pi}{2} - x^2 - \frac{x^6}{6} \right) dx \quad \text{since } |0.4| < 1 \\
 & \approx \left[\frac{\pi}{2}x - \frac{x^3}{3} - \frac{x^7}{42} \right]_0^{0.4} \\
 & \approx 0.607
 \end{aligned}$$



ERRATA

Mathematics: Analysis and Approaches HL

First edition - 2019 first reprint

The following erratum was made on 12/Nov/2020

page 810 **ANSWERS REVIEW SET 2B**, question **9 b**, should read:

- 9 a** $f(x)$: y -intercept 2, $g(x)$: y -intercept -1
b $\frac{5}{2} - \frac{1}{2}\sqrt{5}$ units

The following erratum was made on 27/Jul/2020

page 906 **ANSWERS EXERCISE 28C.1**, question **2 c**, should read:

- 2 c** The times may be affected by:
- weather conditions
 - walking speed
 - physical fitness
 - traffic.

The following errata were made on 17/Jun/2020

page 61 **CHAPTER 2 INVESTIGATION 2**, question **5**, should read:

- 5** For continuous growth, $u_n = u_0 e^{rt}$ where u_0 is the initial amount, r is the annual percentage rate, and t is the number of years.
 Use this formula to find the final amount if \$1000 is invested for **1 year** at a fixed rate of 6% per annum, where the interest is paid continuously.

page 661 **CHAPTER 24 SECTION B**, last line of summary table, should read:

SUMMARY OF IMPORTANT MACLAURIN SERIES

Function	Maclaurin series	Interval of convergence
e^x	$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$	$x \in \mathbb{R}$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$x \in \mathbb{R}$
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$x \in \mathbb{R}$
$\arctan x$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$ x \leq 1$
$\ln(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$-1 < x \leq 1$
$(1+x)^p, p \in \mathbb{R}$	$1 + \sum_{k=1}^{\infty} \frac{p(p-1)\dots(p-k+1)}{k!} x^k$	$ x < 1$

page 700 **EXERCISE 25H**, question **4**, should read:

- 4** Let $y(x) = 1 + \sum_{n=1}^{\infty} \frac{p(p-1)\dots(p-n+1)x^n}{n!}$ for $|x| < 1$. You may assume the series is **convergent** on this interval.

page 895 **ANSWERS EXERCISE 25D**, question **5 f**, should read:

5 **a** $y = \sqrt[3]{\frac{9}{2}x^2 + 1}$ **b** $y = \frac{1}{36}(x - 26)^2$
c $y = e^{x + \frac{1}{3}x^3}$ **d** $y = \arcsin\left(\frac{3}{2}x^2 - \frac{3}{2}\right)$
e $y = \left(\frac{9}{2}\sin 2x + 3\sqrt{3}\right)^{\frac{2}{3}}$
f $y = \ln\left[\sqrt[4]{2x^2 + 4x + 1}\right](e^2 + 3) - 3$

The following errata were made on 04/Jun/2020

page 778 **EXERCISE 28B**, question **15 b**, should read:

- 15 b** Suppose we wish to find the distribution of the random variable $Y = g(X)$ where g is an **increasing and** invertible function.
- i** Show that $F_Y(y) = F_X(g^{-1}(y))$.
- ii** Show that $f_Y(y) = f_X(g^{-1}(y)) \times \frac{d}{dy} g^{-1}(y)$.

page 894 **ANSWERS REVIEW SET 24B**, question **2 a**, was altered in error. It should read as originally printed:

2 a

n	2	4	6
$M_n(0.1)$	-0.2506	-0.2506	-0.2506
$M_n(1)$	-0.3125	-0.3281	-0.3320
$M_n(2)$	-0.5	-0.75	-1

page 907 **ANSWERS EXERCISE 28D.1**, question **10 b**, was altered in error. It should read as originally printed:

10 a $\approx 84.1\%$ **b** ≈ 0.880

The following errata were made on 13/May/2020

page 583 **EXERCISE 21F**, question **21**, should read:

21 Find $\int \frac{1}{ax^2 + bx + c} dx$, $a \neq 0$.

Hint: You will need to consider the cases $b^2 < 4ac$, $b^2 > 4ac$, and $b^2 = 4ac$ separately.

page 865 **ANSWERS EXERCISE 18C**, question **4 c**, should read:

- 4** **a** increasing for $x \geq 0$, decreasing for $x \leq 0$
b never increasing, decreasing for all $x \in \mathbb{R}$
c increasing for $x \geq 0$, never decreasing
d increasing for $x \geq -\frac{3}{4}$, decreasing for $x \leq -\frac{3}{4}$

page 873 **ANSWERS REVIEW SET 18C**, question **25 b**, should read:

25 **a** $0 \leq x \leq \frac{\pi}{2}$ and $\frac{3\pi}{2} \leq x \leq 2\pi$
b $f'(x) = -\frac{\sin x}{2\sqrt{\cos x}}$, increasing for $\frac{3\pi}{2} \leq x \leq 2\pi$,
decreasing for $0 \leq x \leq \frac{\pi}{2}$

page 880 **ANSWERS EXERCISE 21B**, question **6 b**, should read:


6 **a** $\frac{4^x}{\ln 4} + c$ **b** $3 \ln |x| - \log_2 |x| + c$
c $\frac{5^x}{\ln 5} - \frac{2 \times 7^x}{\ln 7} + c$ **d** $-\operatorname{cosec} x + c$
e $\sin x - \cot x + c$ **f** $-\frac{8}{3x\sqrt{x}} - \frac{1}{9} \sec x + c$

The following errata were made on 11/May/2020

page 72 **EXERCISE 3B**, question **6**, should read:

- 6** Suppose $\log_a b = x$, $x \neq 0$. Find, in terms of x , the value of $\log_b a$.

page 582 **CHAPTER 21 EXAMPLE 17**, question should read:

Example 17	 Self Tutor
Find $\int \frac{\sqrt{x^2 - 9}}{x} dx$, where $x \geq 3$.	
Let $x = 3 \sec \theta$ $\therefore \frac{dx}{d\theta} = 3 \sec \theta \tan \theta$	

page 582 **EXERCISE 21F**, questions **17 b**, **18 f**, and **18 i**, should read:

17 Find:

a $\int \frac{1}{36 + 4x^2} dx$ using $x = 3 \tan \theta$

b $\int \frac{\sqrt{4x^2 - 1}}{5x} dx$, $x \geq \frac{1}{2}$ using $x = \frac{1}{2} \sec \theta$.

18 Integrate with respect to x :

a $\frac{x^2}{9 + x^2}$

b $\frac{x^2}{\sqrt{1 - x^2}}$

c $\sqrt{9 - x^2}$

d $\frac{4 \ln x}{x(1 + [\ln x]^2)}$

e $x^2 \sqrt{1 - x^2}$

f $\frac{\sqrt{x^2 - 4}}{x}$, $x \geq 2$

g $\frac{1}{\sqrt{9 - 4x^2}}$

h $\frac{1}{x(9 + 4[\ln x]^2)}$

i $\frac{1 - 2x}{\sqrt{4 - x^2}}$

j $\frac{x + 4}{x^2 + 4}$

k $\frac{1}{x(x^2 + 16)}$

l $\frac{3}{x\sqrt{x^2 - 4}}$, $x > 2$

m $\frac{1}{x^2 \sqrt{16 - x^2}}$

n $\frac{1}{x^2 + 2x + 3}$

o $\frac{1}{x(1 + x^2)}$

p $x^2 \sqrt{4 - x^2}$

page 667 **CHAPTER 24 EXAMPLE 6**, solution to part **b** should read:

$$\begin{aligned}
 \text{b } \arccos x - \arccos(0) &= \int_0^x \frac{-1}{\sqrt{1 - t^2}} dt \\
 \therefore \arccos x - \frac{\pi}{2} &= - \int_0^x \left(1 + \sum_{k=1}^{\infty} \frac{(2k)!}{4^k (k!)^2} t^{2k} \right) dt \\
 &= - \left(\int_0^x 1 dt + \sum_{k=1}^{\infty} \frac{(2k)!}{4^k (k!)^2} \int_0^x t^{2k} dt \right) \\
 &= - \left([t]_0^x + \sum_{k=1}^{\infty} \frac{(2k)!}{4^k (k!)^2} \left[\frac{t^{2k+1}}{2k+1} \right]_0^x \right) \\
 &= - \left(x + \sum_{k=1}^{\infty} \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1} \right) \\
 \therefore \arccos x &= \frac{\pi}{2} - x - \sum_{k=1}^{\infty} \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1}
 \end{aligned}$$

This is valid provided $|x| < 1$. This covers the domain of $\arccos x$ except its endpoints ± 1 .

page 814 **ANSWERS EXERCISE 3H**, question **10 a**, should read:

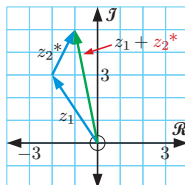
10 a $\frac{x}{e}$, Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y > 0\}$

page 835 **ANSWERS EXERCISE 10A**, question **3 a**, should read:

3 a $2 + 4 + 6 + 8 + 10 + \dots + 2n = n(n + 1)$,
 $\sum_{i=1}^n 2i = n(n + 1)$ for all $n \in \mathbb{Z}^+$.

page 850 **ANSWERS EXERCISE 14A**, question **6 a**, should read:

6 a $-1 + 5i$



page 853 **ANSWERS REVIEW SET 14A**, question **5 c**, should read:

- 5 a** A reflection in the \mathcal{R} -axis, followed by a stretch with scale factor 2.
b A rotation of π about O, followed by a stretch with scale factor $\frac{1}{2}$.
c A reflection in the \mathcal{R} -axis, followed by an anticlockwise rotation of $\frac{\pi}{2}$ about O.

page 854 **ANSWERS EXERCISE 15A**, question **7**, replace with:

7 $\lim_{x \rightarrow a} f(x) = l \Leftrightarrow \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} l$
 $\Leftrightarrow \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} l = 0$
 $\Leftrightarrow \lim_{x \rightarrow a} (f(x) - l) = 0$

page 872 **ANSWERS REVIEW SET 18A**, question **24 c**, should read:

- 24 a** concave up for $x \geq \frac{4}{3}$, concave down for $x \leq \frac{4}{3}$
b concave up for $x \leq -3$,
concave down for $-3 \leq x < 0$ and $x > 0$
c concave up for $-4 < x \leq -2$ and $x > 0$,
concave down for $x < -4$ and $-2 \leq x < 0$

page 881 **ANSWERS EXERCISE 21B**, question **11 d**, should read:

11 d $f(x) = 2x + 3 \arctan x + c$

page 38 **CHAPTER 1 ACTIVITY 3**, question **4** should read:

page 117 **EXERCISE 5C**, questions **13 c** and **14 c**, should read:


- page 150 **REVIEW SET 5B**, question **28 a**, replace with:

- page 186 **EXERCISE 7E**, question **7**, should read:

- Explain why the total number of **ways** in which the tiles can be **ordered** is $\frac{144!}{(4!)^{34}}$.

page 231 **REVIEW SET 9A**, question **10 a**, should read:

- page 238 **CHAPTER 10 EXAMPLE 3**, solution to part **a**, third line should read:

-  If $n = 0$, $4^n + 2 = 4^0 + 2 = 3$ which is divisible by 3.
- $$\begin{aligned} 4^n + 2 &= (1 + 3)^n + 2 \\ &= \binom{n}{0} 3^0 + \binom{n}{1} 3^1 + \binom{n}{2} 3^2 + \binom{n}{3} 3^3 + \dots + \binom{n}{n-1} 3^{n-1} + \binom{n}{n} 3^n + 2 \\ &= 3 + \binom{n}{1} 3 + \binom{n}{2} 3^2 + \binom{n}{3} 3^3 + \dots + \binom{n}{n-1} 3^{n-1} + \binom{n}{n} 3^n \end{aligned}$$

When we perform this process we may obtain one of the forms below, with corresponding numbers of solutions:

$$\begin{array}{ccc} \left(\begin{array}{cc|c} 1 & \square & \square \\ 0 & 1 & \square \end{array} \right) & \left(\begin{array}{cc|c} 1 & \square & \square \\ 0 & 0 & \square \end{array} \right) \leftarrow \text{not zero} & \left(\begin{array}{cc|c} 1 & \square & \square \\ 0 & 0 & 0 \end{array} \right) \\ \text{1 solution} & \text{no solutions} & \text{infinitely many solutions} \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 15 \\ 0 & 1 & -\frac{3}{5} & \frac{23}{5} \\ 0 & 0 & 1 & -1 \end{array} \right) \quad \begin{array}{l} -\frac{1}{5}R_2 \rightarrow R_2 \\ -\frac{1}{17}R_3 \rightarrow R_3 \end{array}$$

Using row 3, $z = -1$

8 Consider the system
$$\begin{cases} x + 4y + mz = -m \\ (m+1)x + 4y + z = 1 \\ 4x + 4y + z = 1 \end{cases} \quad \text{where } m \in \mathbb{R}, m \neq 0.$$

Example 30

Self Tutor

Find **two** vectors of length 3 units which **are** perpendicular to $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \end{pmatrix} = -4 + 4 = 0$$

\therefore vectors of the form $k\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $k \neq 0$ are perpendicular to $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Now $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ has length $\sqrt{16+1} = \sqrt{17}$ units

$$\therefore |k|\sqrt{17} = 3$$

$$\therefore |k| = \frac{3}{\sqrt{17}}$$

$$\therefore k = \pm \frac{3}{\sqrt{17}}$$

\therefore the vectors of length 3 units which are perpendicular to $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

are $\pm \frac{3}{\sqrt{17}} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, which are $\begin{pmatrix} \frac{12}{\sqrt{17}} \\ \frac{3}{\sqrt{17}} \end{pmatrix}$ and $\begin{pmatrix} -\frac{12}{\sqrt{17}} \\ -\frac{3}{\sqrt{17}} \end{pmatrix}$.

8 Find **two** **vectors** of length:

4 ABCD is a quadrilateral in which **P** **bisects both** [AC] and [BD].

page 339 **EXERCISE 13F EXAMPLE 11**, question **4**, third and fourth lines of solution should read:

$$\begin{aligned} &\sim \left(\begin{array}{cc|c} 3 & 5 & 9 \\ 0 & 19 & 57 \end{array} \right) \quad 2R_1 - 3R_2 \rightarrow R_2 \quad \left\{ \begin{array}{ccc} 6 & 10 & 18 \\ -6 & 9 & 39 \\ 0 & 19 & 57 \end{array} \right\} \\ &\sim \left(\begin{array}{cc|c} 1 & \frac{5}{3} & 3 \\ 0 & 1 & 3 \end{array} \right) \quad \frac{1}{3}R_1 \rightarrow R_1 \\ &\quad \frac{1}{19}R_2 \rightarrow R_2 \end{aligned}$$

page 409 **CHAPTER 15 INVESTIGATION**, questions **2 c i** and **ii** become **2 b i** and **ii**, question **2 d** becomes **2 c**.

page 508 **REVIEW SET 18A**, question **4** was a duplicate of Review Set 17A Question 8, change to:

- 4** Find all points on the curve $y = 4x^3 + 6x^2 - 13x + 1$ where the gradient of the tangent is 11.

page 511 **REVIEW SET 18A**, question **32 d**, should read:

- 32** Consider the function $f(x) = xe^{-x^2}$.
- a** Find $f'(x)$ and $f''(x)$.
 - b** Find and classify the turning points of $y = f(x)$.
 - c** Use l'Hôpital's rule to consider the behaviour of $f(x)$ as $x \rightarrow \pm\infty$.
 - d** Find the inflection points of $y = f(x)$.
 - e** Sketch $y = f(x)$, showing the features you have found.

page 607 **CHAPTER 22 ACTIVITY 1**, question **2 a** change to match example diagram:

- 2** Consider the total area enclosed between $y = -x^3 + x^2 + 6x$ and $y = 2x + 4$ on the interval $-2 \leq x \leq 2$.

a Explain why the total area is equal to
$$\int_{-2}^2 |(-x^3 + x^2 + 6x) - (2x + 4)| dx$$
$$= \int_{-2}^2 |-x^3 + x^2 + 4x - 4| dx$$

page 657 **EXERCISE 24A**, question **7**, replace with new questions **7** and **8**:

- 7** Show that the Maclaurin series representation for $\ln\left(\frac{1+x}{1-x}\right)$ is $\sum_{k=1}^{\infty} \frac{2}{2k-1} x^{2k-1}$.
- 8 a** Prove by mathematical induction that
$$\frac{d^n}{dx^n}(\arctan x) = \frac{i(-1)^{n-1}(n-1)!}{2} \left(\frac{1}{(x+i)^n} - \frac{1}{(x-i)^n} \right) \quad \text{for all } n \in \mathbb{Z}^+.$$
- b** Hence show that the Maclaurin series representation for $\arctan x$ is $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} x^{2k-1}$.

page 689 **EXERCISE 25D**, question **20**, should read:

- 20** Since water and oil are *immiscible*, oil spilt in water will form a cylindrical patch on the surface of the water. The radius of the patch increases at a rate proportional to the thickness of the patch, which is the height of the cylinder.

page 760 **CHAPTER 27 INVESTIGATION 2**, question **1**, change for simplicity:

What to do:

- 1** Click on the icon to access the demonstration. It shows the graph of the binomial distribution for $X \sim B(n, p)$. Set $n = 25$ and $p = 0.1$.
- a** What is the mode of X ?
 - b** Describe the shape of the distribution.

page 805 **ANSWERS REVIEW SET 1A**, question **11 c**, $x = 3$ is not a valid solution:

9 $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, \text{ or } 2\pi$ **11 c** $x = \frac{16}{3}$

page 809 **ANSWERS EXERCISE 2F**, question **18**, should read:

18 $e^1 \approx \sum_{k=0}^{19} \frac{1}{k!} 1^k \approx 2.718\,281\,828$

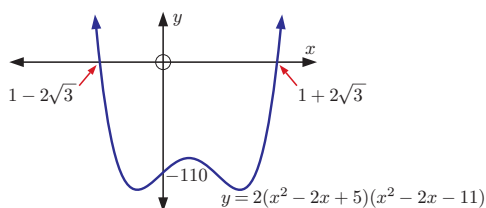
page 818 **ANSWERS EXERCISE 5D**, questions **2 b** and **4**, should read:

2 a $a = 2, b = -2$ or $a = -2, b = 2$
b $a = 2, b = -1$
c $a = 3, b = -1$
4 $a = -2, b = 2, x = -1 \pm \sqrt{3}$

page 821 **ANSWERS REVIEW SET 5A**, question **19 b**, should have diagram:

19 a $m = 1, n = \pm 2$

b



To make room for this diagram, the following questions have moved to subsequent pages:
ANSWERS REVIEW SET 5A questions **24 a** and **b** moved from page 821 to page 822,
ANSWERS REVIEW SET 5B questions **25** and **26** moved from page 822 to page 823,
ANSWERS EXERCISE 6C.1 question **3 a** moved from page 823 to page 824.

page 822 **ANSWERS REVIEW SET 5B**, question **3 b**, should read:

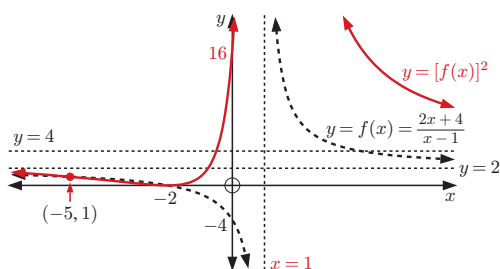
3 a $(3x - 2)$ and $(x - 3)$, zeros are $\frac{2}{3}$ and 3
b $(z - 1)$, $(2z + 1)$, and $(z^2 - 2z + 6)$,
 zeros are 1, $-\frac{1}{2}$, $1 \pm i\sqrt{5}$

page 823 **ANSWERS REVIEW SET 5B**, question **28 a**, should read:

28 a $\frac{1}{3}$ and $\pm i\sqrt{7}$ **b** $-7, -1, \text{ and } 2$

page 823 **ANSWERS EXERCISE 6B**, question **7 d**, should include vertical asymptote label:

7 d



page 831 **ANSWERS EXERCISE 7E**, questions **8 b**, **27 b**, and **27 c**, should read:

8 a $\frac{1}{2}n^2 - \frac{1}{2}n, n \in \mathbb{Z}^+, n \geq 2$
b $\frac{1}{24}n^4 - \frac{1}{4}n^3 + \frac{11}{24}n^2 - \frac{1}{4}n, n \in \mathbb{Z}^+, n \geq 4$
27 a $\binom{52}{13} = 635\,013\,559\,600$
b $\binom{13}{4}\binom{39}{9} = 151\,519\,319\,380$ **c** $\frac{\binom{13}{4}\binom{39}{9}}{\binom{52}{13}} \approx 0.239$

page 832 ANSWERS REVIEW SET 7B, questions 11 a and 12, should read:

11 a 6435 b 2 627 625

12 4347

page 832 ANSWERS EXERCISE 8A, question 4 b, should read:

4 a $1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

b $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

page 834 ANSWERS EXERCISE 9C, questions 6 a and 7 a, should read:

6 a $(a-b)(a+b) = b(a-b) \not\Rightarrow a+b=b$
 $2a = a \not\Rightarrow 2=1$

b $\frac{4x-40}{6-x} = \frac{4x-40}{13-x} \not\Rightarrow 6-x=13-x$

7 a $6x-12 = 3(x-2) \not\Rightarrow 6x-12+3(x-2)=0$

b $x(x-6) = 3(-3) \not\Rightarrow x=3 \vee x-6=-3$

page 834 ANSWERS EXERCISE 9E, question 5, should read:

5 Hint: Let $n = 7k, 7k+1, \dots, 7k+6$, and show that n^2+4 never leaves remainder 0 when divided by 7.

page 835 ANSWERS REVIEW SET 9B, question 4 b, should read:

4 a not equivalent b not equivalent

page 837 ANSWERS EXERCISE 11D, questions 8 c and d, should read:

8 c The system has infinitely many solutions if the last row is all zeros. This occurs when $m = 3$. The solutions have the

form $x = \frac{4+2t}{3}, y = \frac{-13-11t}{12}, z = t$, where $t \in \mathbb{R}$.

d $x = 0, y = \frac{m}{2(m-1)}, z = \frac{m+1}{1-m}, m \in \mathbb{R}, m \neq 1 \text{ or } 3$

page 837 ANSWERS REVIEW SET 11A, question 3 b, should read:

3 a $\left(\begin{array}{cc|c} 4 & -6 & -1 \\ a & 2 & 3 \end{array} \right)$ b $\sim \left(\begin{array}{cc|c} 4 & -6 & -1 \\ -3a & -6 & -9 \end{array} \right)$

page 838 ANSWERS REVIEW SET 11B, questions 1 b, 5 and 8 c, should read:

1 a consistent; $x = \frac{2}{3}, y = 0$ is a solution

b inconsistent; $x+4y+z$ cannot be equal to both 1 and -1 simultaneously.

5 • If $a = -8, b = 20$, there are infinitely many solutions of the form $x = 5 + 2t, y = t$, where $t \in \mathbb{R}$.

• If $a = -8, b \neq 20$, there are no solutions.

• If $a \neq -8$, the system has the unique solution

$$x = 5 + 2\left(\frac{b-20}{a+8}\right), y = \frac{b-20}{a+8}.$$

8 a $\sim \left(\begin{array}{ccc|c} 1 & 4 & -1 & k \\ 0 & 8 & -1 & 3+2k \\ 0 & 0 & k-3 & 3-k \end{array} \right)$

b infinitely many solutions when $k = 3$:

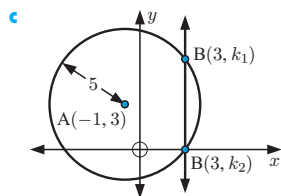
$$x = \frac{t-3}{2}, y = \frac{t+9}{8}, z = t, \text{ where } t \in \mathbb{R}$$

c $x = -2, y = \frac{k+1}{4}, z = -1, k \in \mathbb{R}, k \neq 3$

page 841 **ANSWERS EXERCISE 12I**, question **14 c**, should read:

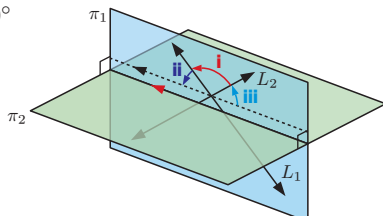
14 a $\vec{AB} = \begin{pmatrix} 4 \\ k-3 \end{pmatrix}$, $|\vec{AB}| = \sqrt{16 + (k-3)^2}$ units

b $k = 0$ or 6



page 847 **ANSWERS EXERCISE 13H**, question **7 e**, diagram should be:

7 e 180°

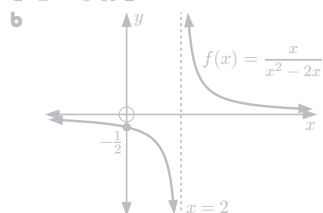


page 851 **ANSWERS EXERCISE 14D.3**, question **7 a**, should read:

7 a $|-z| = 3$, $\arg(-z) = \theta \pm \pi$

page 854 **ANSWERS EXERCISE 15B**, question **4 c iii**, should read:

4 a $x = 0$ or 2

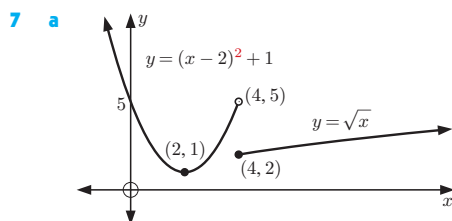


c i does not exist

ii -1

iii $-\frac{1}{2}$

page 857 **ANSWERS REVIEW SET 15B**, question **7 a**, should have correct function label:

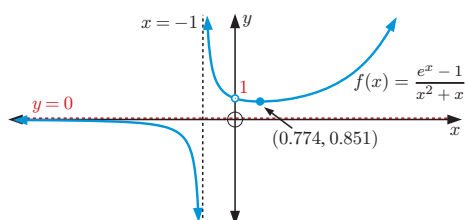


page 859 **ANSWERS EXERCISE 17A**, question **9**, should read:

9 $f'_-(2) = 14 \neq f'_+(2) = 16$
 \therefore not differentiable at $x = 2$.

page 871 **ANSWERS EXERCISE 18H**, questions **5 e** and **10**, should read:

5 e



10 Hint: Consider $\lim_{x \rightarrow \infty} \frac{\ln x}{x^k}$, $k > 0$.

page 879 **ANSWERS EXERCISE 21A**, question **7**, should read:

7 a $\frac{d}{dx}(3^x) = 3^x \ln 3$
 $\therefore \int 3^x \ln 3 \, dx = 3^x + c$
 $\therefore \int 3^x \, dx = \frac{3^x}{\ln 3} + c$

b $\frac{d}{dx}(a^x) = a^x \ln a$
 $\therefore \int a^x \ln a \, dx = a^x + c$
 $\therefore \int a^x \, dx = \frac{a^x}{\ln a} + c, a > 0, a \neq 1$

page 881 **ANSWERS EXERCISE 21D**, question **9 b iii**, should read:

9 b i $\frac{3^{2x-1}}{2 \ln 3} + c$ **ii** $-\frac{5^{-x}}{\ln 5} + c$
iii $\frac{2^{5x}}{5 \ln 2} + \frac{7^{1-2x}}{2 \ln 7} + c$

page 883 **ANSWERS EXERCISE 21F**, question **21**, should read:

21 $\frac{2}{\sqrt{4ac-b^2}} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + d$ if $b^2 < 4ac$,
 $\frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + d$ if $b^2 > 4ac$,
 $-\frac{2}{2ax+b} + d$ if $b^2 = 4ac$.

page 883 **ANSWERS REVIEW SET 21A**, questions **16 f** and **17 b**, should read:

16 a $\frac{1}{2} \ln |x^2 + 4x| + c$ **b** $e^{x^2-1} + c$
c $\frac{1}{10} \sin^{10} x + c$ **d** $-\frac{1}{2} \ln |\cos 2x| + c$
e $e^{\sin x} + c$ **f** $\frac{1}{2}(\arcsin x)^2 + c$

17 a $-\frac{32}{3}(4-x)^{\frac{3}{2}} + \frac{16}{5}(4-x)^{\frac{5}{2}} - \frac{2}{7}(4-x)^{\frac{7}{2}} + c$
b $-x - 6 \ln |2-x| - \frac{12}{2-x} + \frac{4}{(2-x)^2} + c$
c $\frac{2}{3}(x+2)^{\frac{3}{2}} - (x+2) - 2\sqrt{x+2} + 2 \ln(\sqrt{x+2} + 1) + c$

page 885 **ANSWERS EXERCISE 22C**, question **12 f**, should read:

12 a $\frac{1}{2} \text{ units}^2$ **b** $(e-1) \text{ units}^2$ **c** $4\frac{1}{2} \text{ units}^2$
d 18 units^2 **e** $\left(2e - \frac{2}{e}\right) \text{ units}^2$ **f** $\frac{3^{\ln 3} - 1}{\ln 3} \text{ units}^2$
g $\frac{\pi}{6} \text{ units}^2$ **h** $4 \arctan \frac{1}{2} \text{ units}^2$

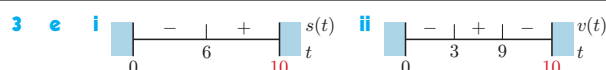
page 887 **ANSWERS EXERCISE 22I**, question **2 c**, should read:

2 a $\frac{1}{4} \text{ units}^2$ **b** $\frac{\pi}{2} \text{ units}^2$ **c** $\frac{1}{\ln 3} \text{ units}^2$

page 888 **ANSWERS REVIEW SET 22B**, question **17 a**, should read:

17 a $3(\ln 3 + \ln 2 - 1) \text{ units}^2$ **b** $\left(3\sqrt[3]{4} - \frac{3}{4}\right) \text{ units}^2$

page 888 **ANSWERS EXERCISE 23B.1**, question **3 e**, sign diagrams should terminate at $t = 10$:



- 3 c** At $t = 5$ s, the stone is 367.5 m above the ground and moving upward at 49 m s^{-1} . **It has acceleration -9.8 m s^{-2} .**
At $t = 12$ s, the stone is 470.4 m above the ground and moving downward at 19.6 m s^{-1} . **It has acceleration -9.8 m s^{-2} .**

5 c i $t > \frac{1}{2}$ ii $0 \leq t < \frac{1}{2}$ e $\frac{4}{25} \text{ cm s}^{-2}$

5 a $\sin(x + \frac{\pi}{2}) = \sum_{k=0}^{\infty} \frac{(-1)^k (x + \frac{\pi}{2})^{2k+1}}{(2k+1)!}$

b $\sin(x + \frac{\pi}{2})$
 $= (\frac{\pi}{2} + x) - \frac{1}{3!}(\frac{\pi}{2} + x)^3 + \frac{1}{5!}(\frac{\pi}{2} + x)^5$
 $- \frac{1}{7!}(\frac{\pi}{2} + x)^7 + \dots$
 $= \frac{\pi}{2} + x - \frac{1}{3!}((\frac{\pi}{2})^3 + 3(\frac{\pi}{2})^2x + 3(\frac{\pi}{2})x^2 + x^3)$
 $+ \frac{1}{5!}((\frac{\pi}{2})^5 + 5(\frac{\pi}{2})^4x + 10(\frac{\pi}{2})^3x^2 + 10(\frac{\pi}{2})^2x^3$
 $+ 5(\frac{\pi}{2})x^4 + x^5)$
 $- \frac{1}{7!}((\frac{\pi}{2})^7 + 7(\frac{\pi}{2})^6x + 21(\frac{\pi}{2})^5x^2 + 35(\frac{\pi}{2})^4x^3$
 $+ 35(\frac{\pi}{2})^3x^4 + 21(\frac{\pi}{2})^2x^5 + 7(\frac{\pi}{2})x^6 + x^7)$
 $+ \dots$
 $= (\frac{\pi}{2} - \frac{1}{3!}(\frac{\pi}{2})^3 + \frac{1}{5!}(\frac{\pi}{2})^5 - \frac{1}{7!}(\frac{\pi}{2})^7 + \dots)$
 $+ (1 - \frac{1}{2!}(\frac{\pi}{2})^2 + \frac{1}{4!}(\frac{\pi}{2})^4 - \frac{1}{6!}(\frac{\pi}{2})^6 + \dots)x$
 $- \frac{1}{2!}(\frac{\pi}{2} - \frac{1}{3!}(\frac{\pi}{2})^3 + \frac{1}{5!}(\frac{\pi}{2})^5 - \dots)x^2$
 $- \frac{1}{3!}(1 - \frac{1}{2!}(\frac{\pi}{2})^2 + \frac{1}{4!}(\frac{\pi}{2})^4 - \dots)x^3$
 $+ \frac{1}{4!}(\frac{\pi}{2} - \frac{1}{3!}(\frac{\pi}{2})^3 + \dots)x^4 + \dots$
 Now, $\frac{\pi}{2} - \frac{1}{3!}(\frac{\pi}{2})^3 + \frac{1}{5!}(\frac{\pi}{2})^5 - \frac{1}{7!}(\frac{\pi}{2})^7 + \dots = \sin \frac{\pi}{2} = 1$
 and $1 - \frac{1}{2!}(\frac{\pi}{2})^2 + \frac{1}{4!}(\frac{\pi}{2})^4 - \frac{1}{6!}(\frac{\pi}{2})^6 + \dots = \cos \frac{\pi}{2} = 0$
 $\therefore \sin(x + \frac{\pi}{2}) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots = \cos x$
 This result agrees with the identity $\sin(x + \frac{\pi}{2}) = \cos x$.

4 a $e^{-x^3} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{3k}}{k!} = 1 - x^3 + \frac{x^6}{2!} - \frac{x^9}{3!} + \dots$

b $e^{-x^3} \cos x = 1 - \frac{1}{2}x^2 - x^3 + \frac{1}{24}x^4 + \frac{1}{2}x^5 + \frac{359}{720}x^6$
 $- \frac{1}{24}x^7 - \frac{10\,079}{40\,320}x^8 - \frac{119}{720}x^9 - \dots$

3 a i $e^{2x} - 1 = \sum_{k=1}^{\infty} \frac{2^k x^k}{k!} = 2x + 2x^2 + \frac{4}{3}x^3 + \dots$

ii $\sin 4x = \sum_{k=0}^{\infty} \frac{(-1)^k 4^{2k+1} x^{2k+1}}{(2k+1)!}$
 $= 4x - \frac{32}{3}x^3 + \frac{128}{15}x^5 - \dots$

7 a $\sin x$ has zeros $n\pi$, $n \in \mathbb{Z}$.
 $\frac{\sin x}{x}$ has zeros $n\pi$, $n \in \mathbb{Z}$, $n \neq 0$.

b $\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$ for all $x \in \mathbb{R}$, $x \neq 0$

page 894 **ANSWERS REVIEW SET 24B**, questions **2 a** and **8 a i**, should read:

2 a

n	2	4	6
$M_n(0.1)$	-0.2506	-0.2506	-0.2506
$M_n(1)$	-0.3281	-0.3330	-0.3333
$M_n(2)$	-0.75	-1.25	1.75

*This erratum for **2 a** was made in error, please disregard it.*

8 a i $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \dots$

page 895 **ANSWERS EXERCISE 25D**, questions **2 e** and **2 f**, should read:

2 e $Q = Ae^{2t} - \frac{3}{2}$

f $Q = -\frac{3}{2} \pm \sqrt{t+c}$

page 896 **ANSWERS EXERCISE 25E**, question **2 d ii**, should read:

2 d i ≈ 924 rodents **ii** ≈ 23.0 years **iii** 3000 rodents

page 896 **ANSWERS EXERCISE 25F**, question **4**, should read:

4 $x = \pm\sqrt{2}e^{-\frac{\pi}{4}}$ **5 b** $y = -x \ln\left(\frac{1}{x} + c\right)$

page 897 **ANSWERS REVIEW SET 25B**, questions **7 a**, **11 a**, and **11 b**, should read:

7 a $y = (\frac{1}{2}x + 2)^2$ **b** $y = e^{\sin x - 3}$ **8** $y = \frac{(x+2)^2}{25(x-2)}$

11 a $\frac{dV}{dt} = k\sqrt{h}$ **b** $V = 2 \times 2 \times h$, $\frac{dh}{dt} = \frac{k}{4}\sqrt{h}$

page 907 **ANSWERS EXERCISE 28D.2**, question **10 b**, should read:

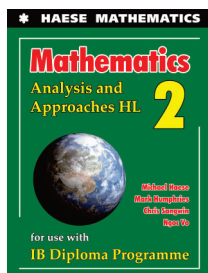
10 a $\approx 84.1\%$ **b** $\approx \cancel{0.979}$ *This erratum for **10 b** was made in error, please disregard it.*

The following erratum was made on 22/Oct/2019

page 13 **USEFUL FORMULAE, COUNTING AND THE BINOMIAL THEOREM**, first dot point should read:

COUNTING AND THE BINOMIAL THEOREM

- $\binom{n}{r} = \begin{cases} \frac{n!}{r!(n-r)!}, & n \in \mathbb{Z}^+, r \in \mathbb{N}, r \leq n \\ \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}, & n \in \mathbb{Q}, r \in \mathbb{Z}^+, \text{ and } \binom{n}{0} = 1 \end{cases}$



ERRATA

Mathematics: Analysis and Approaches HL

First edition - 2019 initial print

The following errata were made on 19/Sep/2019

page 285 **EXERCISE 12E**, question **7**, part **i** was removed.

page 329 **EXERCISE 13B**, question **2**, should read:

- 2** Show that the lines $L_1: x = 2 + 5p, y = 19 - 3p, z = 9 + 2p, p \in \mathbb{R}$
and $L_2: x = 3 + 4r, y = 7 + 10r, z = -3 + 5r, r \in \mathbb{R}$ are perpendicular.

page 361 **REVIEW SET 13A**, question **6 a ii**, should read:

- a** Find, in terms of **i** and **j**, the:
- i** initial position vector of the yacht
 - ii** **velocity** vector of the yacht
 - iii** position vector of the yacht after t hours, $t \geq 0$.

page 572 **CHAPTER 21 SECTION D**, blue box should read:

$$\therefore \int (ax + b)^n dx = \frac{1}{a} \frac{(ax + b)^{n+1}}{(n+1)} + c \text{ for } n \neq -1, a \neq 0.$$

page 593 **EXERCISE 22A**, question **16 c**, should read:

- 16 c** Solve $\int_0^a x \sec^2 x dx = \frac{3}{4}, 0 < a < \frac{\pi}{2}$ directly using technology.

page 614 **EXERCISE 22G.2**, question **1 a**, should read:

- 1** Find the volume of the solid formed when the following are revolved through 2π about the y -axis:
- a** $y = x^2, x \geq 0$, between $y = 0$ and $y = 4$ **b** $y = \sqrt{x}$ between $y = 1$ and $y = 4$

page 614 **EXERCISE 22G.2**, question **3**, should read:

- 3** A wooden bowl is made in the shape of a *paraboloid* by revolving the curve $y = \frac{1}{4}x^2, x \geq 0$, between $y = 0$ and $y = 4$ through 2π about the y -axis. Find the capacity of the bowl.

page 628 **REVIEW SET 22B**, question **22**, should read:

- 22** Over the course of a day, the **rate of** solar energy being transferred into Callum's solar panels is given by $E(t) = 2 \sin\left(\frac{t-5}{5}\right) + \frac{1}{2} \sin\left(\frac{t-5}{4}\right)$ kW where t is the time in hours after midnight, $5 \leq t \leq 20$.

page 840 **ANSWERS EXERCISE 12E**, question **7**, part **i** was removed.

page 873 **ANSWERS REVIEW SET 18B**, question **25 b**, should read:

- 25 a** $0 \leq x \leq \frac{\pi}{2}$ and $\frac{3\pi}{2} \leq x \leq 2\pi$
- b** $f'(x) = -\frac{\sin x}{2\sqrt{\cos x}}$, increasing for $\frac{3\pi}{2} < x \leq 2\pi$,
decreasing for $0 \leq x < \frac{\pi}{2}$

page 878 **ANSWERS EXERCISE 20C**, question **2 b**, should read:

- b** The antiderivative of e^{kx} is $\frac{1}{k} e^{kx}$, where $k \neq 0$ is a constant.