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ERRATA

Mathematics: Analysis and Approaches HL

First edition - 2024 fifth reprint

The following erratum was made on 12/Sep/2025

page 888 ANSWERS REVIEW SET 22B, question 17a, should read:

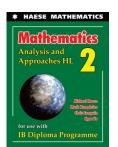
17 a $(6 \ln 3 - 4) \text{ units}^2$

b $(3\sqrt[3]{4} - \frac{3}{4})$ units²

The following erratum was made on 24/Jul/2025

page 871 ANSWERS EXERCISE 18H, question 3, replace with:

3 Repeated use of l'Hôpital's rule does not help as the expression in the limit becomes more and more complicated.



Mathematics: Analysis and Approaches HL

First edition - 2023 fourth reprint

The following errata were made on 05/Sep/2024

page 437 CHAPTER 17 INVESTIGATION 1, question 1, should read:

What to do:

1 Use the binomial expansion $(x+h)^n = \sum_{r=0}^n \binom{n}{r} x^{n-r} h^r$ $= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \dots + \binom{n}{n} h^n$ and the first principles formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = x^n$ for $n \in \mathbb{N}$.

page 654 CHAPTER 24 OPENING PROBLEM, question e, should read:

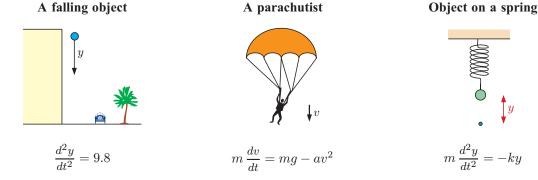
e Can we write an *exact* expression for e^x in the form $e^x = \sum_{k=0}^{\infty} c_k x^k$ where $c_k \in \mathbb{R}$?

page 654 CHAPTER 24 HISTORICAL NOTE, third paragraph, should read:

In the 14th century, the Indian mathematician **Madhava of Sangamagrama** discovered infinite series expansions for a number of trigonometric functions, as well as infinite series for calculating π such as $\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$.

page 674 CHAPTER 24 SECTION A, example diagram for Object on a spring, should read:

For example:



page 700 CHAPTER 25 ACTIVITY 1, replace paragraphs 4 and 5 with:

In the case of a mass on a spring, k is determined by the mass of the object, and the stiffness of the spring.

In the case of a pendulum, k is determined by the weight on its end, and the length of the pendulum.

• The standard deviation of X is $\sigma = \sqrt{\operatorname{Var}(X)} = \sqrt{np(1-p)}$.

The following erratum was made on 04/Jul/2024

page 250 CHAPTER 10 ACTIVITY, 7th line, should read:

Infinite descent was a historical precursor to mathematical induction. However, it is in fact a special form of proof by contradiction.

The following errata were made on 13/Mar/2024

page 133 **EXERCISE 5I**, question **9**, should read:

- 9 f(x) is a polynomial of degree n with leading coefficient 1. The sum of its zeros is 5, and the product of its zeros is -3. Suppose $g(x) = [f(x)]^2$. Find:
 - a the degree of g(x)

- **b** the sum of the zeros of g(x)
- the product of the zeros of g(x)
- d the y-intercept of y = g(x).

page 216 **EXERCISE 9B**, question **8**, should read:

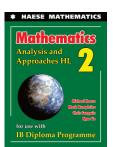
8 Consider a 3-digit number "abc", $a \neq c$. Written backwards, it is "cba". Let S be the 3-digit result when the smaller of the two numbers is subtracted from the larger. When S is written backwards and the result is added to S, prove that the sum is always 1089.

For example: 276 backwards is 672, so S = 396 and 396 + 693 = 1089.

The following erratum was made on 23/Oct/2023

page 530 **EXERCISE 19B**, question **20**, should read:

20 A cone has radius r cm and fixed slant height s cm. Find the ratio of s:r which maximises the volume of a cone.



Mathematics: Analysis and Approaches HL

First edition - 2021 third reprint

The following erratum was made on 17/Apr/2023

page 821 ANSWERS EXERCISE 5M, question 4 f, should read:

```
4 a -2.09 < x < 0.572 or x > 2.51

b x < -1.66 or 0.327 < x < 1.84

c -2.55 < x < 0.290 or x \geqslant 2.26

d x \leqslant \frac{3}{2}

e -1.20 < x < -0.125 or x > 1.33

f x < 0.924
```

The following erratum was made on 15/Mar/2023

page 433 REVIEW SET 16A, question 9, should read:

9 Let
$$f(x) = \begin{cases} 11 - 2x, & x \ge 3 \\ x + 2, & x < 3 \end{cases}$$
.

a Sketch the function y = f(x).

b Is f differentiable at x = 3?

The following errata were made on 06/Dec/2022

page 116 **EXERCISE 5C**, question **8 b**, should read:

- 8 The graph of the polynomial P(x) has three x-intercepts, α , β , and γ .
 - **a** State the values of $P(\alpha)$, $P(\beta)$, and $P(\gamma)$.
 - **b** Explain why P(x) cannot have a real linear factor (x-a) such that $a \neq \alpha, \beta$, or γ .

page 817 ANSWERS EXERCISE 5C, question 8 b, should read:

- **8** a $P(\alpha) = 0$, $P(\beta) = 0$, $P(\gamma) = 0$
- **b** If P(x) has a real linear factor of (x-a), then P(a)=0 which implies that P(x) has x-intercept a which is distinct from the x-intercepts of α , β , and γ . But P(x) has only three x-intercepts and since

But P(x) has only three x-intercepts and since $a \neq \alpha$, β , or γ , a cannot be one of them.

 \therefore P(x) cannot have a factor of (x-a).

page 386 **CHAPTER 14 HISTORICAL NOTE**, second paragraph should read:

HISTORICAL NOTE

EULER'S BEAUTIFUL EQUATION

One of the most remarkable results in mathematics is known as Euler's beautiful equation $e^{i\pi}=-1$ named after Leonhard Euler.

It is called beautiful because it links together three great constants of mathematics: Euler's number e, the imaginary number i, and the ratio of a circle's circumference to its diameter π .

Harvard lecturer **Benjamin Pierce** said of $e^{i\pi} = -1$,

"Gentlemen, that is surely true, it is absolutely paradoxical; we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth."

Mathematics Analysis and Approaches HL 2

ERRATA

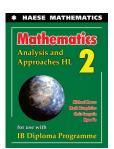
Mathematics: Analysis and Approaches HL

First edition - 2020 second reprint

The following erratum was made on 12/Nov/2020

page 667 **CHAPTER 24 EXAMPLE 6**, solution to part **c** should read:

$$\begin{array}{ll} \bullet & \arccos x \approx \frac{\pi}{2} - x - \frac{1}{6}x^3 \quad \{ \text{using the first 3 terms} \} \\ & \therefore \quad \arccos(x^2) \approx \frac{\pi}{2} - x^2 - \frac{1}{6}x^6 \quad \text{for} \quad |x| < 1 \\ & \therefore \quad \int_0^{0.4} \arccos\left(x^2\right) \, dx \approx \int_0^{0.4} \left(\frac{\pi}{2} - x^2 - \frac{x^6}{6}\right) dx \quad \text{since} \quad |0.4| < 1 \\ & \approx \left[\frac{\pi}{2}x - \frac{x^3}{3} - \frac{x^7}{42}\right]_0^{0.4} \end{array}$$



Mathematics: Analysis and Approaches HL

First edition - 2019 first reprint

The following erratum was made on 12/Nov/2020

page 810 ANSWERS REVIEW SET 2B, question 9 b, should read:

- 9 a f(x): y-intercept 2, g(x): y-intercept -1
 - **b** $\frac{5}{2} \frac{1}{2}\sqrt{5}$ units

The following erratum was made on 27/Jul/2020

page 906 ANSWERS EXERCISE 28C.1, question 2 c, should read:

- The times may be affected by:
 - weather conditions
 - · walking speed
 - · physical fitness
 - traffic.

The following errata were made on 17/Jun/2020

page 61 CHAPTER 2 INVESTIGATION 2, question 5, should read:

- **5** For continuous growth, $u_n = u_0 e^{rt}$ where u_0 is the initial amount, r is the annual percentage rate, and t is the number of years.
 - Use this formula to find the final amount if \$1000 is invested for 1 year at a fixed rate of 6% per annum, where the interest is paid continuously.

page 661 **CHAPTER 24 SECTION B**, last line of summary table, should read:

SUMMARY OF IMPORTANT MACLAURIN SERIES

Function	Maclaurin series	Interval of convergence
e^x	$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$	$x \in \mathbb{R}$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$x \in \mathbb{R}$
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$x \in \mathbb{R}$
$\arctan x$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$ x \leqslant 1$
$\ln(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$-1 < x \leqslant 1$
$(1+x)^p, \ p \in \mathbb{R}$	$1 + \sum_{k=1}^{\infty} \frac{p(p-1)(p-k+1)}{k!} x^k$	x < 1

page 700 EXERCISE 25H, question 4, should read:

4 Let $y(x) = 1 + \sum_{n=1}^{\infty} \frac{p(p-1)....(p-n+1)x^n}{n!}$ for |x| < 1. You may assume the series is convergent on this interval.

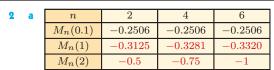
5 **a**
$$y = \sqrt[3]{\frac{9}{2}x^2 + 1}$$
 b $y = \frac{1}{36}(x - 26)^2$
c $y = e^{x + \frac{1}{3}x^3}$ **d** $y = \arcsin\left(\frac{3}{2}x^2 - \frac{3}{2}\right)$
e $y = \left(\frac{9}{2}\sin 2x + 3\sqrt{3}\right)^{\frac{2}{3}}$
f $y = \ln\left[\sqrt[4]{|2x^2 + 4x + 1|}\left(e^2 + 3\right) - 3\right]$

The following errata were made on 04/Jun/2020

page 778 **EXERCISE 28B**, question **15 b**, should read:

- Suppose we wish to find the distribution of the random variable Y = g(X) where g is an increasing and invertible function.
 - **i** Show that $F_Y(y) = F_X(g^{-1}(y))$.
 - ii Show that $f_Y(y) = f_X(g^{-1}(y)) \times \frac{d}{du} g^{-1}(y)$.

page 894 ANSWERS REVIEW SET 24B, question 2 a, was altered in error. It should read as originally printed:



page 907 ANSWERS EXERCISE 28D.1, question 10 b, was altered in error. It should read as originally printed:

10 a $\approx 84.1\%$ $b \approx 0.880$

The following errata were made on 13/May/2020

page 583 EXERCISE 21F, question 21, should read:

21 Find
$$\int \frac{1}{ax^2 + bx + c} dx$$
, $a \neq 0$.

Hint: You will need to consider the cases $b^2 < 4ac$, $b^2 > 4ac$, and $b^2 = 4ac$ separately.

page 865 ANSWERS EXERCISE 18C, question 4 c, should read:

- **a** increasing for $x \ge 0$, decreasing for $x \le 0$
 - **b** never increasing, decreasing for all $x \in \mathbb{R}$
 - c increasing for $x \ge 0$, never decreasing
 - **d** increasing for $x \geqslant -\frac{3}{4}$, decreasing for $x \leqslant -\frac{3}{4}$

page 873 ANSWERS REVIEW SET 18C, question 25 b, should read:

25 a
$$0 \leqslant x \leqslant \frac{\pi}{2}$$
 and $\frac{3\pi}{2} \leqslant x \leqslant 2\pi$

b
$$f'(x) = -\frac{\sin x}{2\sqrt{\cos x}}$$
, increasing for $\frac{3\pi}{2} \leqslant x \leqslant 2\pi$, decreasing for $0 \leqslant x \leqslant \frac{\pi}{2}$

page 880 ANSWERS EXERCISE 21B, question 6 b, should read:

6 a
$$\frac{4^{3}}{\ln 4} + c$$

b
$$3 \ln |x| - \log_2 |x| + \epsilon$$

a
$$\frac{4^x}{\ln 4} + c$$
 b $3\ln|x| - \log_2|x| + c$ **c** $\frac{5^x}{\ln 5} - \frac{2 \times 7^x}{\ln 7} + c$ **d** $-\csc x + c$ **e** $\sin x - \cot x + c$ **f** $-\frac{8}{3x\sqrt{x}} - \frac{1}{9}\sec x + c$

$$\mathbf{d} - \csc x + c$$

$$e \sin x - \cot x + c$$

$$\int \frac{8}{2\pi \sqrt{\pi}} - \frac{1}{9} \sec x +$$

The following errata were made on 11/May/2020

page 72 **EXERCISE 3B**, question **6**, should read:

6 Suppose $\log_a b = x$, $x \neq 0$. Find, in terms of x, the value of $\log_b a$.

page 582 CHAPTER 21 EXAMPLE 17, question should read:

Find $\int \frac{\sqrt{x^2-9}}{x} \, dx$, where $x \geqslant 3$. Let $x=3\sec\theta$ \therefore $\frac{dx}{d\theta}=3\sec\theta\tan\theta$

page 582 EXERCISE 21F, questions 17 b, 18 f, and 18 l, should read:

17 Find:

a
$$\int \frac{1}{36 + 4x^2} \, dx$$
 using $x = 3 \tan \theta$ **b** $\int \frac{\sqrt{4x^2 - 1}}{5x} \, dx$, $x \geqslant \frac{1}{2}$ using $x = \frac{1}{2} \sec \theta$.

18 Integrate with respect to x:

Integrate with respect to
$$x$$
:

a $\frac{x^2}{9+x^2}$
b $\frac{x^2}{\sqrt{1-x^2}}$
c $\sqrt{9-x^2}$
d $\frac{4 \ln x}{x(1+[\ln x]^2)}$
e $x^2\sqrt{1-x^2}$
f $\frac{\sqrt{x^2-4}}{x}$, $x \geqslant 2$
g $\frac{1}{\sqrt{9-4x^2}}$
h $\frac{1}{x(9+4[\ln x]^2)}$
i $\frac{1-2x}{\sqrt{4-x^2}}$
j $\frac{x+4}{x^2+4}$
k $\frac{1}{x(x^2+16)}$
l $\frac{3}{x\sqrt{x^2-4}}$, $x > 2$
m $\frac{1}{x^2\sqrt{16-x^2}}$
n $\frac{1}{x^2+2x+3}$
o $\frac{1}{x(1+x^2)}$
p $x^2\sqrt{4-x^2}$

page 667 CHAPTER 24 EXAMPLE 6, solution to part **b** should read:

b
$$\arccos x - \arccos(0) = \int_0^x \frac{-1}{\sqrt{1 - t^2}} dt$$

 $\therefore \arccos x - \frac{\pi}{2} = -\int_0^x \left(1 + \sum_{k=1}^\infty \frac{(2k)!}{4^k (k!)^2} t^{2k}\right) dt$
 $= -\left(\int_0^x 1 dt + \sum_{k=1}^\infty \frac{(2k)!}{4^k (k!)^2} \int_0^x t^{2k} dt\right)$
 $= -\left([t]_0^x + \sum_{k=1}^\infty \frac{(2k)!}{4^k (k!)^2} \left[\frac{t^{2k+1}}{2k+1}\right]_0^x\right)$
 $= -\left(x + \sum_{k=1}^\infty \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1}\right)$
 $\therefore \arccos x = \frac{\pi}{2} - x - \sum_{k=1}^\infty \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1}$

This is valid provided |x| < 1. This covers the domain of $\arccos x$ except its endpoints +1.

10 a
$$\frac{x}{e}$$
, Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y > 0\}$

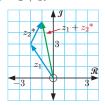
page 835 ANSWERS EXERCISE 10A, question 3 a, should read:

3 a
$$2+4+6+8+10+\ldots+2n=n(n+1),$$

$$\sum_{i=1}^n \frac{2i}{n}=n(n+1) \quad \text{for all} \quad n\in\mathbb{Z}^+.$$

page 850 ANSWERS EXERCISE 14A, question 6 a, should read:





page 853 ANSWERS REVIEW SET 14A, question 5 c, should read:

- **5** a A reflection in the \Re -axis, followed by a stretch with scale factor 2.
 - **b** A rotation of π about O, followed by a stretch with scale factor $\frac{1}{2}$.
 - **c** A reflection in the \mathcal{R} -axis, followed by an anticlockwise rotation of $\frac{\pi}{2}$ about O.

page 854 ANSWERS EXERCISE 15A, question 7, replace with:

7
$$\lim_{x \to a} f(x) = l \Leftrightarrow \lim_{x \to a} f(x) = \lim_{x \to a} l$$

 $\Leftrightarrow \lim_{x \to a} f(x) - \lim_{x \to a} l = 0$
 $\Leftrightarrow \lim_{x \to a} (f(x) - l) = 0$

page 872 ANSWERS REVIEW SET 18A, question 24 c, should read:

- **24** a concave up for $x \geqslant \frac{4}{3}$, concave down for $x \leqslant \frac{4}{3}$
 - **b** concave up for $x \le -3$,

concave down for $-3 \le x < 0$ and x > 0

concave up for $-4 < x \leqslant -2$ and x > 0, concave down for x < -4 and $-2 \leqslant x < 0$

page 881 ANSWERS EXERCISE 21B, question 11 d, should read:

11 d
$$f(x) = 2x + 3 \arctan x + c$$

The following errata were made on 27/Apr/2020

page 38 CHAPTER 1 ACTIVITY 3, question 4 should read:

4 Predict the graph of $f(x) = \sum_{k=1}^{\infty} \frac{\sin[(2k-1)x]}{2k-1}$.

page 117 EXERCISE 5C, questions 13 c and 14 c, should read:

- 13 a Suppose a quadratic function with *rational* coefficients has the *irrational* zero $p + q\sqrt{n}$. Prove that the other zero must be the radical conjugate $p q\sqrt{n}$.
 - **b** $1-\sqrt{2}$ is a zero of x^2+ax+b where $a,b\in\mathbb{Q}$. Find a and b.
 - Find all real quadratic functions with rational coefficients such that one zero is $2 + \sqrt{5}$.
- 14 a Prove that if a *real* quadratic function has one zero p + qi, then the other zero must be the complex conjugate p qi.
 - **b** 3+i is a zero of x^2+ax+b where $a, b \in \mathbb{R}$. Find a and b.
 - Find all real quadratic functions such that one zero is $\sqrt{2} + i$.
 - **d** a + ai is a zero of $x^2 + 4x + b$ where $a, b \in \mathbb{R}$. Find a and b.

page 150 REVIEW SET 5B, question 28 a, replace with:

- **28** Find all zeros of:
 - a $3z^3 z^2 + 21z 7$

b $x^3 + 6x^2 - 9x - 14$

page 186 **EXERCISE 7E**, question **7**, should read:

- 7 A Mahjong set contains:
 - 4 of each of 9 tiles, in each of 3 suits
- 4 of each of 4 winds

• 4 of each of 3 dragons

• 1 of each of 4 flowers

• 1 of each of 4 seasons.

Explain why the total number of ways in which the tiles can be ordered is $\frac{144!}{(4!)^{34}}$.

page 231 REVIEW SET 9A, question 10 a, should read:

- **10** a Prove that the fifth powers of the numbers k = 0, 1, 2, 3, ..., 9 all have last digit k.
 - **b** Hence prove that n always has the same last digit as its 5th power n^5 , for all $n \in \mathbb{Z}$. **Hint:** Consider writing an integer in the form 10m + k.

page 238 CHAPTER 10 EXAMPLE 3, solution to part a, third line should read:

a If n = 0, $4^n + 2 = 4^0 + 2 = 3$ which is divisible by 3. $4^n + 2 = (1+3)^n + 2$ $= {n \choose 0} 3^0 + {n \choose 1} 3^1 + {n \choose 2} 3^2 + {n \choose 3} 3^3 + \dots + {n \choose n-1} 3^{n-1} + {n \choose n} 3^n + 2$ $= 3 + {n \choose 1} 3 + {n \choose 2} 3^2 + {n \choose 3} 3^3 + \dots + {n \choose n-1} 3^{n-1} + {n \choose n} 3^n$ When we perform this process we may obtain one of the forms below, with corresponding numbers of

page 262 CHAPTER 11 EXAMPLE 4, fifth line should read:

$$\sim \begin{pmatrix} 1 & 3 & -1 & 15 \\ 0 & 1 & -\frac{3}{5} & \frac{23}{5} \\ 0 & 0 & 1 & -1 \end{pmatrix} \qquad -\frac{1}{5}R_2 \to R_2$$
$$-\frac{1}{17}R_3 \to R_3$$

Using row 3, z = -1

page 265 EXERCISE 11D, question 8, first line should read

8 Consider the system $\begin{cases} x+4y+mz=-m\\ (m+1)x+4y+z=1\\ 4x+4y+z=1 \end{cases} \text{ where } m\in\mathbb{R},\ m\neq 0.$

page 304 CHAPTER 12 EXAMPLE 30, should read:

Example 30

Self Tutor

Find two vectors of length 3 units which are perpendicular to $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

$$\begin{pmatrix} -1\\4 \end{pmatrix} \bullet \begin{pmatrix} 4\\1 \end{pmatrix} = -4 + 4 = 0$$

 \therefore vectors of the form $k \binom{4}{1}$, $k \neq 0$ are perpendicular to $\binom{-1}{4}$. Now $\binom{4}{1}$ has length $\sqrt{16+1} = \sqrt{17}$ units

$$\therefore |k|\sqrt{17} = 3$$

$$|k| = \frac{3}{\sqrt{17}}$$

$$\therefore k = \pm \frac{3}{\sqrt{17}}$$

 \therefore the vectors of length 3 units which are perpendicular to $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

are
$$\pm \frac{3}{\sqrt{17}} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$
, which are $\begin{pmatrix} \frac{12}{\sqrt{17}} \\ \frac{3}{\sqrt{17}} \end{pmatrix}$ and $\begin{pmatrix} -\frac{12}{\sqrt{17}} \\ -\frac{3}{\sqrt{17}} \end{pmatrix}$.

page 304 EXERCISE 12L, question 8, first line should read:

8 Find two vectors of length:

page 308 EXERCISE 12M, question 4, first line should read:

4 ABCD is a quadrilateral in which P bisects both [AC] and [BD].

page 339 EXERCISE 13F EXAMPLE 11, question 4, third and fourth lines of solution should read:

$$\sim \begin{pmatrix} 3 & 5 & 9 \\ 0 & 19 & 57 \end{pmatrix} 2R_1 - 3R_2 \to R_2 \longrightarrow \begin{cases} 6 & 10 & 18 \\ -6 & 9 & 39 \\ \hline 0 & 19 & 57 \end{cases} \\
\sim \begin{pmatrix} 1 & \frac{5}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix} \frac{\frac{1}{3}R_1 \to R_1}{\frac{1}{19}R_2 \to R_2}$$

page 409 CHAPTER 15 INVESTIGATION, questions 2 c i and ii become 2 b i and ii, question 2 d becomes 2 c.

page 508 REVIEW SET 18A, question 4 was a duplicate of Review Set 17A Question 8, change to:

4 Find all points on the curve $y = 4x^3 + 6x^2 - 13x + 1$ where the gradient of the tangent is 11.

page 511 REVIEW SET 18A, question 32 d, should read:

- **32** Consider the function $f(x) = xe^{-x^2}$.
 - **a** Find f'(x) and f''(x).
 - **b** Find and classify the turning points of y = f(x).
 - Use l'Hôpital's rule to consider the behaviour of f(x) as $x \to \pm \infty$.
 - **d** Find the inflection points of y = f(x).
 - Sketch y = f(x), showing the features you have found.

page 607 CHAPTER 22 ACTIVITY 1, question 2 a change to match example diagram:

- **2** Consider the total area enclosed between $y = -x^3 + x^2 + 6x$ and y = 2x + 4 on the interval $-2 \le x \le 2$.
 - **a** Explain why the total area is equal to $\int_{-2}^{2} \left| (-x^3 + x^2 + 6x) (2x + 4) \right| dx$ $= \int_{-2}^{2} \left| -x^3 + x^2 + 4x 4 \right| dx$

page 657 EXERCISE 24A, question 7, replace with new questions 7 and 8:

- 7 Show that the Maclaurin series representation for $\ln\left(\frac{1+x}{1-x}\right)$ is $\sum_{k=1}^{\infty}\frac{2}{2k-1}x^{2k-1}$.
- 8 a Prove by mathematical induction that

$$\frac{d^n}{dx^n}(\arctan x) = \frac{i(-1)^{n-1}(n-1)!}{2} \left(\frac{1}{(x+i)^n} - \frac{1}{(x-i)^n} \right) \text{ for all } n \in \mathbb{Z}^+.$$

b Hence show that the Maclaurin series representation for $\arctan x$ is $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} x^{2k-1}$.

page 689 **EXERCISE 25D**, question **20**, should read:

20 Since water and oil are immiscible, oil spilt in water will form a cylindrical patch on the surface of the water. The radius of the patch increases at a rate proportional to the thickness of the patch, which is the height of the cylinder.

page 760 CHAPTER 27 INVESTIGATION 2, question 1, change for simplicity:

What to do:

- 1 Click on the icon to access the demonstration. It shows the graph of the binomial distribution for $X \sim B(n, p)$. Set n = 25 and p = 0.1.
 - **a** What is the mode of X?
 - **b** Describe the shape of the distribution.

9
$$x=0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, \text{ or } 2\pi$$
 11 c $x=\frac{16}{3}$

$$x = \frac{16}{9}$$

page 809 ANSWERS EXERCISE 2F, question 18, should read:

18
$$e^1 \approx \sum_{k=0}^{19} \frac{1}{k!} 1^k \approx 2.718281828$$

page 818 ANSWERS EXERCISE 5D, questions 2 b and 4, should read:

2 a
$$a=2$$
, $b=-2$ or $a=-2$, $b=2$

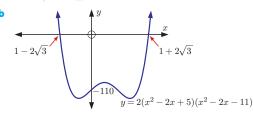
b
$$a = 2, b = -1$$

4
$$a = 3, b = -1$$

4 $a = -2, b = 2, x = -1 \pm \sqrt{3}$

page 821 ANSWERS REVIEW SET 5A, question 19 b, should have diagram:

19 **a** $m = 1, n = \pm 2$



To make room for this diagram, the following questions have moved to subsequent pages:

ANSWERS REVIEW SET 5A questions 24 a and b moved from page 821 to page 822,

ANSWERS REVIEW SET 5B questions 25 and 26 moved from page 822 to page 823,

ANSWERS EXERCISE 6C.1 question 3 a moved from page 823 to page 824.

page 822 ANSWERS REVIEW SET 5B, question 3 b, should read:

3 a
$$(3x-2)$$
 and $(x-3)$, zeros are $\frac{2}{3}$ and 3

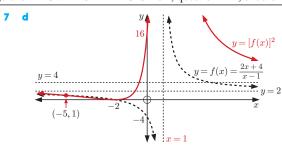
b
$$(z-1)$$
, $(2z+1)$, and (z^2-2z+6) , zeros are $1, -\frac{1}{2}, 1 \pm i\sqrt{5}$

page 823 ANSWERS REVIEW SET 5B, question 28 a, should read:

28 a
$$\frac{1}{3}$$
 and $\pm i\sqrt{7}$

b
$$-7$$
, -1 , and 2

page 823 ANSWERS EXERCISE 6B, question 7 d, should include vertical asymptote label:



page 831 ANSWERS EXERCISE 7E, questions 8 b, 27 b, and 27 c, should read:

8 a
$$\frac{1}{2}n^2 - \frac{1}{2}n$$
, $n \in \mathbb{Z}^+$, $n \geqslant 2$

b
$$\frac{1}{24}n^4 - \frac{1}{4}n^3 + \frac{11}{24}n^2 - \frac{1}{4}n, \ n \in \mathbb{Z}^+, \ n \geqslant 4$$

27 a
$$\binom{52}{13} = 635013559600$$

b
$$\binom{13}{4}\binom{39}{9} = 151519319380$$
 c $\frac{\binom{13}{4}\binom{39}{9}}{\binom{52}{13}} \approx 0.239$

11 a 6435

b 2627625

12 4347

page 832 ANSWERS EXERCISE 8A, question 4 b, should read:

4 a 1 5 10 10 5 1
b
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

page 834 ANSWERS EXERCISE 9C, questions 6 a and 7 a, should read:

6 **a**
$$(a-b)(a+b) = b(a-b) \implies a+b=b$$

2a = a $\implies 2=1$

b
$$\frac{4x-40}{6-x} = \frac{4x-40}{13-x} \implies 6-x = 13-x$$

7 **a**
$$6x - 12 = 3(x - 2) \implies 6x - 12 + 3(x - 2) = 0$$

b
$$x(x-6) = 3(-3) \implies x = 3 \lor x - 6 = -3$$

page 834 ANSWERS EXERCISE 9E, question 5, should read:

5 Hint: Let n = 7k, 7k+1, ..., 7k+6, and show that n^2+4 never leaves remainder 0 when divided by 7.

page 835 ANSWERS REVIEW SET 9B, question 4 b, should read:

4 a not equivalent b not equivalent

page 837 ANSWERS EXERCISE 11D, questions 8 c and d, should read:

8 • The system has infinitely many solutions if the last row is all zeros. This occurs when m=3. The solutions have the

$$\text{form } x=\frac{4+2t}{3}, \ y=\frac{-13-11t}{12}, \ z=t, \ \text{where } \ t\in \mathbb{R}.$$

d
$$x = 0, \ y = \frac{m}{2(m-1)}, \ z = \frac{m+1}{1-m}, \ m \in \mathbb{R}, \ m \neq 1 \text{ or } 3$$

page 837 ANSWERS REVIEW SET 11A, question 3 b, should read:

3 a
$$\begin{pmatrix} 4 & -6 & -1 \\ a & 2 & 3 \end{pmatrix}$$
 b $\sim \begin{pmatrix} 4 & -6 & -1 \\ -3a & -6 & -9 \end{pmatrix}$

page 838 ANSWERS REVIEW SET 11B, questions 1 b, 5 and 8 c, should read:

- 1 a consistent; $x = \frac{2}{3}$, y = 0 is a solution
 - **b** inconsistent; x + 4y + z cannot be equal to both 1 and -1 simultaneously.
- **5** If a=-8, b=20, there are infinitely many solutions of the form x=5+2t, y=t, where $t \in \mathbb{R}$.
 - If a = -8, $b \neq 20$, there are no solutions.
 - If $a \neq -8$, the system has the unique solution

$$x = 5 + 2\left(\frac{b-20}{a+8}\right), \ \ y = \frac{b-20}{a+8}.$$

b infinitely many solutions when k = 3:

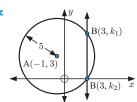
$$x = \frac{t-3}{2}$$
, $y = \frac{t+9}{8}$, $z = t$, where $t \in \mathbb{R}$

$$x = -2, \ y = \frac{k+1}{4}, \ z = -1, \ k \in \mathbb{R}, \ k \neq 3$$

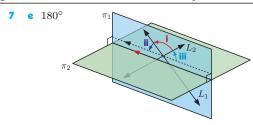
page 841 **ANSWERS EXERCISE 12I**, question **14 c**, should read:

14 a
$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ k-3 \end{pmatrix}$$
, $|\overrightarrow{AB}| = \sqrt{16 + (k-3)^2}$ units

b
$$k = 0$$
 or 6



page 847 ANSWERS EXERCISE 13H, question 7 e, diagram should be:



page 851 **ANSWERS EXERCISE 14D.3**, question **7 a**, should read:

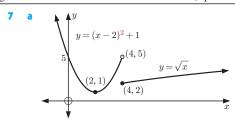
7 **a**
$$|-z| = 3$$
, $arg(-z) = \theta \pm \pi$

page 854 **ANSWERS EXERCISE 15B**, question **4 c iii**, should read:

4 a
$$x = 0$$
 or 2 **b** $f(x) = \frac{x}{x^2 - 2x}$

iii
$$-\frac{1}{2}$$

page 857 ANSWERS REVIEW SET 15B, question 7 a, should have correct function label:

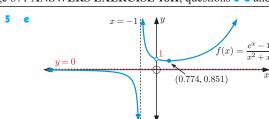


page 859 **ANSWERS EXERCISE 17A**, question **9**, should read:

9
$$f'_{-}(2) = 14 \neq f'_{+}(2) = 16$$

∴ not differentiable at $x = 2$.

page 871 **ANSWERS EXERCISE 18H**, questions **5 e** and **10**, should read:



10 Hint: Consider
$$\lim_{x\to\infty} \frac{\ln x}{x^k}$$
, $k>0$.

7 a
$$\frac{d}{dx}(3^{x}) = 3^{x} \ln 3$$

$$\therefore \int 3^{x} \ln 3 \, dx = 3^{x} + c$$

$$\therefore \int 3^{x} \, dx = \frac{3^{x}}{\ln 3} + c$$
b
$$\frac{d}{dx}(a^{x}) = a^{x} \ln a$$

$$\therefore \int a^{x} \ln a \, dx = a^{x} + c$$

$$\therefore \int a^{x} \, dx = \frac{a^{x}}{\ln a} + c, \ a > 0, \ a \neq 1$$

page 881 ANSWERS EXERCISE 21D, question 9 b iii, should read:

9 **b** i
$$\frac{3^{2x-1}}{2\ln 3} + c$$
 ii $-\frac{5^{-x}}{\ln 5} + c$ iii $\frac{2^{5x}}{5\ln 2} + \frac{7^{1-2x}}{2\ln 7} + c$

page 883 ANSWERS EXERCISE 21F, question 21, should read:

page 883 ANSWERS REVIEW SET 21A, questions 16 f and 17 b, should read:

16 a
$$\frac{1}{2} \ln \left| x^2 + 4x \right| + c$$
 b $e^{x^2 - 1} + c$
c $\frac{1}{10} \sin^{10} x + c$ d $-\frac{1}{2} \ln \left| \cos 2x \right| + c$
e $e^{\sin x} + c$ f $\frac{1}{2} (\arcsin x)^2 + c$
17 a $-\frac{32}{3} (4 - x)^{\frac{3}{2}} + \frac{16}{5} (4 - x)^{\frac{5}{2}} - \frac{2}{7} (4 - x)^{\frac{7}{2}} + c$
b $-x - 6 \ln \left| 2 - x \right| - \frac{12}{2 - x} + \frac{4}{(2 - x)^2} + c$
c $\frac{2}{3} (x + 2)^{\frac{3}{2}} - (x + 2) - 2\sqrt{x + 2} + 2 \ln(\sqrt{x + 2} + 1) + c$

page 885 ANSWERS EXERCISE 22C, question 12 f, should read:

12 a
$$\frac{1}{2}$$
 units² **b** $(e-1)$ units² **c** $4\frac{1}{2}$ units²
d 18 units^2 **e** $\left(2e-\frac{2}{e}\right)$ units²
f $\frac{3\ln 3-1}{\ln 3}$ units²
g $\frac{\pi}{6}$ units² **h** $4 \arctan \frac{1}{2}$ units²

page 887 ANSWERS EXERCISE 22I, question 2 c, should read:

2 a
$$\frac{1}{4}$$
 units² b $\frac{\pi}{2}$ units² c $\frac{1}{\ln 3}$ units²

page 888 ANSWERS REVIEW SET 22B, question 17 a, should read:

17 a
$$3(\ln 3 + \ln 2 - 1)$$
 units² **b** $\left(3\sqrt[3]{4} - \frac{3}{4}\right)$ units²

page 888 **ANSWERS EXERCISE 23B.1**, question 3e, sign diagrams should terminate at t = 10:

3 e i
$$+$$
 $s(t)$ ii $+$ $v(t)$ t 0 0 0 0 0 0 0 0

- 3 c At t = 5 s, the stone is 367.5 m above the ground and moving upward at 49 m s⁻¹. It has acceleration -9.8 m s⁻². At t = 12 s, the stone is 470.4 m above the ground and moving downward at 19.6 m s⁻¹. It has acceleration -9.8 m s⁻².
- 5 c i $t > \frac{1}{2}$ ii $0 \leqslant t < \frac{1}{2}$ e $\frac{4}{25}$ cm s $^{-2}$

page 893 ANSWERS EXERCISE 24C, question 5 b, replace with:

$$\begin{array}{lll} \textbf{5} & \textbf{a} & \sin(x+\frac{\pi}{2}) = \sum\limits_{k=0}^{\infty} \frac{(-1)^k (x+\frac{\pi}{2})^{2k+1}}{(2k+1)!} \\ \textbf{b} & \sin(x+\frac{\pi}{2}) \\ & = (\frac{\pi}{2}+x) - \frac{1}{3!} (\frac{\pi}{2}+x)^3 + \frac{1}{5!} (\frac{\pi}{2}+x)^5 \\ & - \frac{1}{7!} (\frac{\pi}{2}+x)^7 + \dots \\ & = \frac{\pi}{2} + x - \frac{1}{3!} ((\frac{\pi}{2})^3 + 3(\frac{\pi}{2})^2 x + 3(\frac{\pi}{2}) x^2 + x^3) \\ & + \frac{1}{5!} ((\frac{\pi}{2})^5 + 5(\frac{\pi}{2})^4 x + 10(\frac{\pi}{2})^3 x^2 + 10(\frac{\pi}{2})^2 x^3 \\ & + 5(\frac{\pi}{2}) x^4 + x^5) \\ & - \frac{1}{7!} ((\frac{\pi}{2})^7 + 7(\frac{\pi}{2})^6 x + 21(\frac{\pi}{2})^5 x^2 + 35(\frac{\pi}{2})^4 x^3 \\ & + 35(\frac{\pi}{3})^3 x^4 + 21(\frac{\pi}{2})^2 x^5 + 7(\frac{\pi}{2}) x^6 + x^7) \\ & + \dots \\ & = (\frac{\pi}{2} - \frac{1}{3!} (\frac{\pi}{2})^3 + \frac{1}{5!} (\frac{\pi}{2})^5 - \frac{1}{7!} (\frac{\pi}{2})^7 + \dots) \\ & + (1 - \frac{1}{2!} (\frac{\pi}{2})^2 + \frac{1}{4!} (\frac{\pi}{2})^4 - \frac{1}{6!} (\frac{\pi}{2})^6 + \dots) x^2 \\ & - \frac{1}{3!} (1 - \frac{1}{2!} (\frac{\pi}{2})^2 + \frac{1}{4!} (\frac{\pi}{2})^4 - \dots) x^3 \\ & + \frac{1}{4!} (\frac{\pi}{2} - \frac{1}{3!} (\frac{\pi}{2})^3 + \frac{1}{5!} (\frac{\pi}{2})^5 - \frac{1}{7!} (\frac{\pi}{2})^7 + \dots = \sin \frac{\pi}{2} = 1 \\ & \text{and} \quad 1 - \frac{1}{2!} (\frac{\pi}{2})^2 + \frac{1}{4!} (\frac{\pi}{2})^4 - \frac{1}{6!} (\frac{\pi}{2})^6 + \dots = \cos \frac{\pi}{2} = 0 \\ & \therefore \quad \sin(x + \frac{\pi}{2}) = 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \dots = \cos x \end{array}$$

page 893 ANSWERS EXERCISE 24F, question 4, should read:

This result agrees with the identity $\sin(x + \frac{\pi}{2}) = \cos x$.

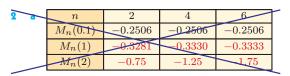
$$\begin{array}{ll} \textbf{4} & \textbf{a} \ e^{-x^3} = \sum\limits_{k=0}^{\infty} \frac{(-1)^k x^{3k}}{k!} = 1 - x^3 + \frac{x^6}{2!} - \frac{x^9}{3!} + \dots \\ \\ \textbf{b} & e^{-x^3} \cos x = 1 - \frac{1}{2} x^2 - x^3 + \frac{1}{24} x^4 + \frac{1}{2} x^5 + \frac{359}{720} x^6 \\ & - \frac{1}{24} x^7 - \frac{10079}{40320} x^8 - \frac{119}{720} x^9 - \dots \\ \end{array}$$

page 894 ANSWERS EXERCISE 24G, questions 3 a ii and 7 b, should read:

3 a i
$$e^{2x} - 1 = \sum_{k=1}^{\infty} \frac{2^k x^k}{k!} = 2x + 2x^2 + \frac{4}{3}x^3 + \dots$$

ii $\sin 4x = \sum_{k=0}^{\infty} \frac{(-1)^k 4^{2k+1} x^{2k+1}}{(2k+1)!}$
 $= 4x - \frac{32}{3}x^3 + \frac{128}{15}x^5 - \dots$

- 7 **a** $\sin x$ has zeros $n\pi$, $n \in \mathbb{Z}$. $\frac{\sin x}{x}$ has zeros $n\pi$, $n \in \mathbb{Z}$, $n \neq 0$. $\sin x$ x^2 x^4 x^6
 - **b** $\frac{\sin x}{x} = 1 \frac{x^2}{3!} + \frac{x^4}{5!} \frac{x^6}{7!} + \dots$ for all $x \in \mathbb{R}, x \neq 0$



This erratum for **2 a** was made in error, please disregard it.

8 a i
$$\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \dots$$

page 895 ANSWERS EXERCISE 25D, questions 2 e and 2 f, should read:

2 •
$$Q = Ae^{2t} - \frac{3}{2}$$
 f $Q = -\frac{3}{2} \pm \sqrt{t+c}$

$$Q = -\frac{3}{2} \pm \sqrt{t+e}$$

page 896 ANSWERS EXERCISE 25E, question 2 d ii, should read:

2 d i
$$\approx 924$$
 rodents ii ≈ 23.0 years iii 3000 rodents

page 896 ANSWERS EXERCISE 25F, question 4, should read:

4
$$x = \pm \sqrt{2}e^{-\frac{\pi}{4}}$$

4
$$x = \pm \sqrt{2}e^{-\frac{\pi}{4}}$$
 5 b $y = -x \ln\left(\frac{1}{x} + c\right)$

page 897 ANSWERS REVIEW SET 25B, questions 7 a, 11 a, and 11 b, should read:

7 **a**
$$y = (\frac{1}{2}x + 2)^2$$

7 **a**
$$y = (\frac{1}{2}x + 2)^2$$
 b $y = e^{\sin x - 3}$ 8 $y = \frac{(x+2)^2}{25(x-2)}$

11 a
$$\frac{dV}{dt} = k\sqrt{k}$$

11 a
$$\frac{dV}{dt} = k\sqrt{h}$$
 b $V = 2 \times 2 \times h$, $\frac{dh}{dt} = \frac{k}{4}\sqrt{h}$

page 907 **ANSWERS EXERCISE 28D.2**, question **10 b**, should read:

10 a
$$\approx 84.1\%$$

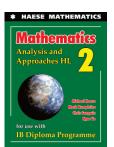
b \approx 1872 This erratum for **10 b** was made in error, please disregard it.

The following erratum was made on 22/Oct/2019

page 13 USEFUL FORMULAE, COUNTING AND THE BINOMIAL THEOREM, first dot point should read:

COUNTING AND THE BINOMIAL THEOREM

$$\bullet \quad {n \choose r} = \begin{cases} \frac{n!}{r!(n-r)!}, & n \in \mathbb{Z}^+, \ r \in \mathbb{N}, \ r \leqslant n \\ \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}, & n \in \mathbb{Q}, \ r \in \mathbb{Z}^+, \ \text{and} \quad {n \choose 0} = 1 \end{cases}$$



Mathematics: Analysis and Approaches HL

First edition - 2019 initial print

The following errata were made on 19/Sep/2019

page 285 EXERCISE 12E, question 7, part i was removed.

page 329 **EXERCISE 13B**, question 2, should read:

Show that the lines L_1 : x=2+5p, y=19-3p, z=9+2p, $p\in\mathbb{R}$ and L_2 : x=3+4r, y=7+10r, z=-3+5r, $r\in\mathbb{R}$ are perpendicular.

page 361 REVIEW SET 13A, question 6 a ii, should read:

- **a** Find, in terms of i and j, the:
 - i initial position vector of the yacht
 - ii velocity vector of the yacht
 - iii position vector of the yacht after t hours, $t \ge 0$.

page 572 CHAPTER 21 SECTION D, blue box should read:

$$\therefore \int (ax+b)^n \ dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{(n+1)} + c \quad \text{for } n \neq -1, \ a \neq 0.$$

page 593 **EXERCISE 22A**, question **16 c**, should read:

16 • Solve $\int_0^a x \sec^2 x \, dx = \frac{3}{4}$, $0 < a < \frac{\pi}{2}$ directly using technology.

page 614 EXERCISE 22G.2, question 1 a, should read:

- 1 Find the volume of the solid formed when the following are revolved through 2π about the y-axis:
 - **a** $y=x^2$, $x \ge 0$, between y=0 and y=4 **b** $y=\sqrt{x}$ between y=1 and y=4

page 614 EXERCISE 22G.2, question 3, should read:

3 A wooden bowl is made in the shape of a *paraboloid* by revolving the curve $y = \frac{1}{4}x^2$, $x \ge 0$, between y = 0 and y = 4 through 2π about the y-axis. Find the capacity of the bowl.

page 628 REVIEW SET 22B, question 22, should read:

Over the course of a day, the rate of solar energy being transferred into Callum's solar panels is given by $E(t) = 2\sin\left(\frac{t-5}{5}\right) + \frac{1}{2}\sin\left(\frac{t-5}{4}\right)$ kW where t is the time in hours after midnight, $5 \le t \le 20$.

page 840 ANSWERS EXERCISE 12E, question 7, part i was removed.

page 873 ANSWERS REVIEW SET 18B, question 25 b, should read:

25 a
$$0\leqslant x\leqslant \frac{\pi}{2}$$
 and $\frac{3\pi}{2}\leqslant x\leqslant 2\pi$
b $f'(x)=-\frac{\sin x}{2\sqrt{\cos x}}$, increasing for $\frac{3\pi}{2}< x\leqslant 2\pi$, decreasing for $0\leqslant x<\frac{\pi}{2}$

page 878 ANSWERS EXERCISE 20C, question 2 b, should read:

b The antiderivative of e^{kx} is $\frac{1}{k}e^{kx}$, where $k \neq 0$ is a