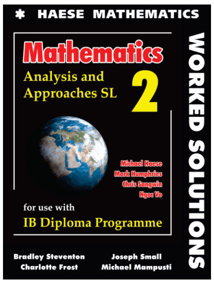


ERRATA

Mathematics: Analysis and Approaches SL

WORKED SOLUTIONS

First edition - 2020

The following errata were made on 25/Jun/2020

page 214 **INVESTIGATION 2** Question **5**, should read:

5 $u_n = u_0 e^{rt}, \;\; u_0 = 1000, \;\; r = 0.06, \;\; t = 1$

$\therefore \;\; u_n = 1000 \times e^{0.06 \times 1} \approx 1061.84$

The final amount is $1061.84.

page 260 **INVESTIGATION 3** Question **2 c**, replace entirely with:

2 **c** From **b**, $f = 261.6 \times 2^n$

$$\therefore \;\; n = \log_2\left(\frac{f}{261.6}\right)$$

Let f_P and f_Q be the frequencies of notes P and Q respectively, where Q is one note above P. There are 12 notes in an octave, and they are evenly spaced on the logarithmic scale.

$$\therefore \;\; \log_2\left(\frac{f_Q}{261.6}\right) - \log_2\left(\frac{f_P}{261.6}\right) = \frac{1}{12}$$

$$\therefore \;\; \log_2\left(\frac{f_Q}{f_P}\right) = \frac{1}{12}$$

$$\therefore \;\; \frac{f_Q}{f_P} = 2^{\frac{1}{12}}$$

So, the ratio between the frequencies of two consecutive notes is $2^{\frac{1}{12}} : 1 \approx 1.06 : 1$.

The following errata were made on 20/May/2020

page 613 **REVIEW SET 13B** Question **20 b**, should read:

20 **b** $f(x) = \sqrt{\cos x} = (\cos x)^{\frac{1}{2}}$

$\therefore \quad f'(x) = \frac{1}{2}(\cos x)^{-\frac{1}{2}}(-\sin x)$

$\qquad = -\dfrac{\sin x}{2\sqrt{\cos x}}$

From **a**, since $f(x)$ is only defined when $0 \leqslant x \leqslant \frac{\pi}{2}$ and $\frac{3\pi}{2} \leqslant x \leqslant 2\pi$, we only consider these values of x.

When $0 \leqslant x < \frac{\pi}{2}$, $f'(x) \leqslant 0$

When $\frac{3\pi}{2} < x \leqslant 2\pi$, $f'(x) \geqslant 0$

$\therefore$ $f(x)$ is increasing for $\frac{3\pi}{2} \leqslant x \leqslant 2\pi$, and decreasing for $0 \leqslant x \leqslant \frac{\pi}{2}$.

page 666 **EXERCISE 15D** Question **5 e**, should read:

5 **e** $\dfrac{d}{dx}F(x) = f(x)$ and $\dfrac{d}{dx}G(x) = g(x)$

$\therefore \quad \dfrac{d}{dx}[F(x) + G(x)] = f(x) + g(x)$

$\therefore$ $F(x) + G(x)$ is the antiderivative of $f(x) + g(x)$.

So, $\displaystyle\int_a^b [f(x) + g(x)]\,dx$

$= [F(b) + G(b)] - [F(a) + G(a)]$

$= [F(b) - F(a)] + [G(b) - G(a)]$

$= \displaystyle\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

page 673 **EXERCISE 16A** Question **1 b**, line 3 should read:

1 **b** $\dfrac{d}{dx}(x^{\frac{3}{2}}) = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

$\therefore \quad \displaystyle\int \tfrac{3}{2}\sqrt{x}\,dx = x^{\frac{3}{2}} + c$

$\therefore \quad \tfrac{3}{2}\displaystyle\int \sqrt{x}\,dx = x^{\frac{3}{2}} + c$

$\therefore \quad \displaystyle\int \sqrt{x}\,dx = \tfrac{2}{3}x^{\frac{3}{2}} + c$

page 674 **EXERCISE 16A** Question **6 b**, should read:

6 **a** $\dfrac{d}{dx}[F(x)+G(x)] = F'(x)+G'(x)$
$= f(x)+g(x)$

b Using **a**, $\displaystyle\int [f(x)+g(x)]\,dx = F(x)+G(x)+c$
$\displaystyle= \int f(x)\,dx + \int g(x)\,dx$

page 684 **EXERCISE 16C** Question **7 c**, line 10 should read:

7 **c** ....

But $f(0)=3$, so $-\cos 0 + d = 3$
$\therefore\ -1+d=3$
$\therefore\ d=4$

$\therefore\ f(x) = -\cos x - x + 4$

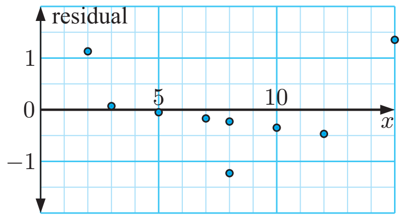
page 817 **EXERCISE 19D** Question **3 b**, should read:

3 **b** There is a strong, negative correlation between *petrol price* and the *number of customers*.

page 831 **ACTIVITY 3** Question **2 a**, text for residual plots **C** and **D** should read:

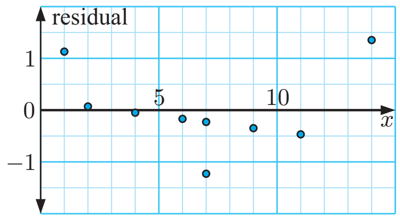
2 **a** ....

C



Most of the residuals in this plot are below the x-axis, with only three residuals above it. The scatter plot shows that only three data values are above the regression line, and the values on the x-axis correspond to those on the scatter diagram.
So **C** is the correct residual plot.

D



This is very similar to residual plot **C**, except the values on the x-axis do not correspond to those on the scatter diagram.
So **D** is not the correct residual plot.

page 839 **EXERCISE 19E** Question **4 b**, should read:

4 **b** The regression line of y against x is $y = ax + b$.

The regression line of x against y is $y = \dfrac{1}{m}x - \dfrac{c}{m}$.

In the regression of y against x, when $x = \overline{x}$,
$$\begin{aligned} y &= a\overline{x} + b \\ &= a\overline{x} + \overline{y} - a\overline{x} \\ &= \overline{y} \end{aligned}$$

and in the regression of x against y, when $y = \overline{y}$,
$$\begin{aligned} x &= m\overline{y} + c \\ &= m\overline{y} + \overline{x} - m\overline{y} \\ &= \overline{x} \end{aligned}$$

So, both lines pass through the mean point $(\overline{x}, \overline{y})$, which means the lines will be the same if and only if their gradients are equal.

$\therefore$ the regression lines will be the same if $a = \dfrac{1}{m}$

$\therefore$ $am = 1$

$\therefore$ $r^2 = 1$ {using **a**}

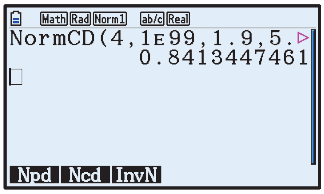
page 897 **INVESTIGATION 3** Question **10**, part **b** calculator screenshot should be:

10 **a** Let X units be the drop in blood pressure of a randomly selected patient.

$X \sim \text{N}(5.9, 1.9^2)$

$\therefore$ $\text{P}(X > 4) \approx 0.841\,34 \approx 0.841$

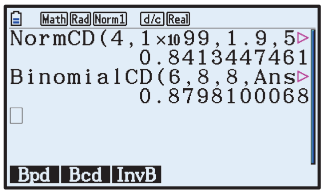
$\therefore$ about 84.1% of patients show a drop of more than 4 units.



b Let Y be the number of patients with a drop of more than 4 units.

$Y \sim \text{B}(8, 0.841\,34)$

$\therefore$ $\text{P}(Y \geqslant 6) \approx 0.880$

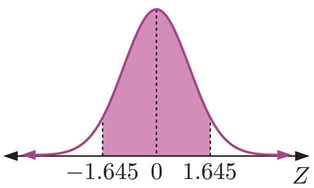


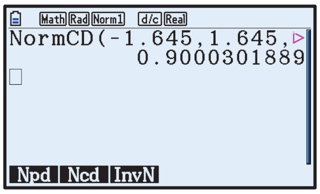
page 898 **EXERCISE 21C.1** Question **1**, should have part **c**:

1 **c** It is reasonable to compare Emma's performances using z-scores as the scores in each of Emma's classes are normally distributed.

page 903 **EXERCISE 21C.2** Question **3 k**, calculator screenshot should be:

3 **k**





$\text{P}(-1.645 \leqslant Z \leqslant 1.645) \approx 0.900$

The following errata were made on 13/May/2020

page 251 **INVESTIGATION 2** Question **2 a**, should read:

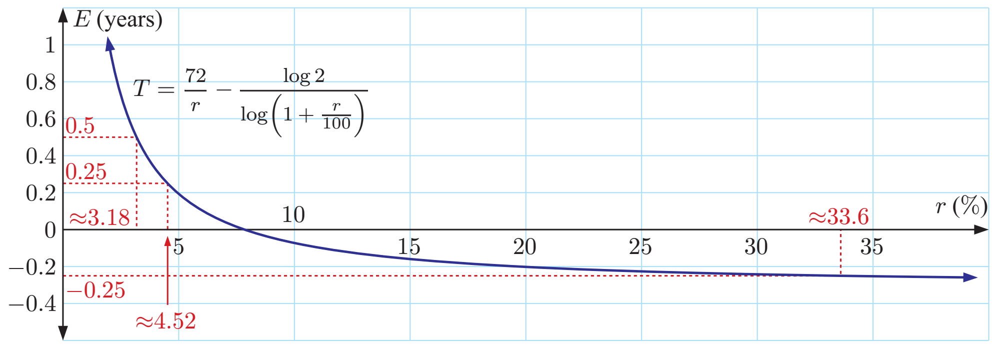
2 Let T be the doubling time.

a For a population growing at 2% per year, we require $(1.02)^T = 2$,

$$\therefore \log((1.02)^T) = \log 2$$
$$\therefore T\log(1.02) = \log 2$$
$$\therefore T = \frac{\log 2}{\log(1.02)} \approx 35.003$$

page 252 **INVESTIGATION 2** Question **3 b** and **c**, replaced with:

3 b The error in estimating the exact doubling time is $E = \dfrac{72}{r} - \dfrac{\log 2}{\log\left(1+\frac{r}{100}\right)}$.



c **i** 6 months is 0.5 years.

From the graph, $E = 0.5$ when $r \approx 3.18$.

$\therefore$ the estimate is accurate to within 6 months for growth rates larger than about 3.18%.

ii 3 months is 0.25 years.

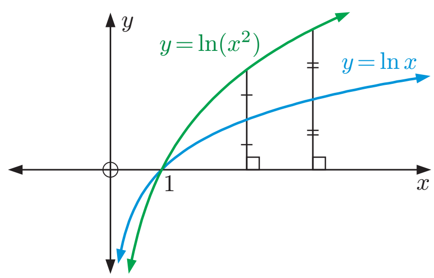
From the graph, $E = 0.25$ when $r \approx 4.52$
and $E = -0.25$ when $r \approx 33.6$

$\therefore$ the estimate is accurate to within 3 months for growth rates between about 4.52% and 33.6%.

page 258 **EXERCISE 6H** Question **4**, first line should read:

4 $y = \ln(x^2)$, $x > 0$
$= 2\ln x$ $\{m\ln b = \ln(b^m)\}$

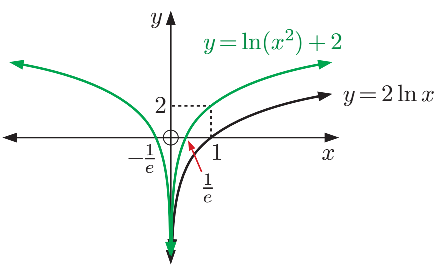
So, the y-values are twice as large for $y = \ln(x^2)$ as they are for $y = \ln x$. Therefore, yes, Kelly is correct.



page 259 **EXERCISE 6H** Question **5 e**, replaced with:

5 e For $x > 0$, $y = \ln(x^2) + 2 = 2\ln x + 2$ is a vertical translation of $y = 2\ln x$, 2 units upwards.

For $x < 0$, $y = \ln(x^2) + 2 = \ln((-x)^2) + 2$ is a reflection of $y = \ln(x^2) + 2$, $x > 0$, in the y-axis.



page 451 **INVESTIGATION 1** Question **3 a ii**, sixth line should read:

3 a ii $f(x) = x^3 - 2x^2$

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h)^2 - (x^3 - 2x^2)}{h} \\
&= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2(x^2 + 2xh + h^2) - x^3 + 2x^2}{h} \\
&= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{2x^2} - 4xh - 2h^2 - \cancel{x^3} + \cancel{2x^2}}{h} \\
&= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 4x - 2h)}{\cancel{h}} \qquad \{\text{as } h \neq 0\} \\
&= \lim_{h \to 0} (3x^2 + 3xh + h^2 - 4x - 2h) \\
&= 3x^2 - 4x
\end{aligned}$$

page 651 **REVIEW SET 14B** Question **2 d**, last line should read:

2 d $P''(t) = ...$

$$= \frac{7500e^{-\frac{t}{4}}\left(2e^{-\frac{t}{4}} - 1\right)}{\left(1 + 2e^{-\frac{t}{4}}\right)^3}$$

page 658 **EXERCISE 15B** Question **1 b**, last row should read:

1 b

n	A_L	A_U
5	0.5497	0.7497
10	0.6105	0.7105
50	0.6561	0.6761
100	0.6615	0.6715
500	0.6656	0.6676

The following errata were made on 28/Apr/2020

page 20 **REVIEW SET 1A** Question **9**, should read:

9 $(2x + 3)(x - 2)^6$

$$= (2x+3)\left[x^6 + \binom{6}{1}x^5(-2) + \binom{6}{2}x^4(-2)^2 + ...\right]$$

(1)

(2)

So, the terms containing x^5 are $2\binom{6}{2}(-2)^2x^5$ from (1)

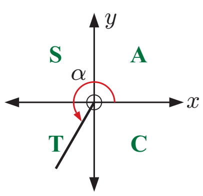
and $3\binom{6}{1}(-2)x^5$ from (2)

$\therefore$ the coefficient of x^5 is $8\binom{6}{2} - 6\binom{6}{1} = 84$

page 394 **REVIEW SET 9A** Question **9 a**, diagram should show correct quadrant:

9 $\sin\alpha = -\frac{3}{4}$ and $\pi \leqslant \alpha \leqslant \frac{3\pi}{2}$

a α is in quadrant 3, so $\cos\alpha$ is negative.



Now $\cos^2\alpha + \sin^2\alpha = 1$

$\therefore \cos^2\alpha + \frac{9}{16} = 1$

$\therefore \cos^2\alpha = \frac{7}{16}$

$\therefore \cos\alpha = -\frac{\sqrt{7}}{4}$ $\{\cos\alpha < 0\}$

page 460 **CHAPTER 12 ACTIVITY** Question **1 d**, last line should read:

1 **d** Suppose $y = x^4$, and therefore $\dfrac{dy}{dx} = 4x^3$.

To estimate the value of 1.95^4, we let $x = 2$ and $\delta x = -0.05$.

$$\begin{aligned}\text{Now} \quad \delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx 4x^3 \times \delta x \\ &\approx 4 \times 2^3 \times (-0.05) \\ &\approx -1.6\end{aligned}$$

Since $2^4 = 16$, we estimate that $1.95^4 \approx 16 - 1.6 \approx 14.4$.

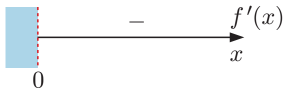
Using technology, $1.95^4 \approx 14.4590$.

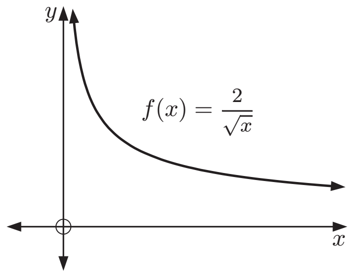
page 545 **EXERCISE 13C** Question **4 e**, should have correct sign diagram:

4 **e** $f(x) = \dfrac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}}$

$\therefore \ f'(x) = -x^{-\frac{3}{2}} = -\dfrac{1}{x\sqrt{x}}$

which has sign diagram:





So, $f(x)$ is only defined for $x > 0$.

$f(x)$ is never increasing, but is decreasing for $x > 0$.

page 703 **REVIEW SET 16A** Question **14**, first line should read:

14 $u = x^2 + \frac{\pi}{3}, \quad \dfrac{du}{dx} = 2x$

page 801 **REVIEW SET 18B** Question **4 c**, should read:

4 **c** $\begin{aligned} x(2) &= 3(2) - 2\sqrt{2} \\ &= 6 - 2\sqrt{2} \\ &\approx 3.17 \text{ cm}\end{aligned}$ $\quad \begin{aligned} v(2) &= 3 - \tfrac{3}{2}\sqrt{2} \\ &\approx 0.879 \text{ cm s}^{-1}\end{aligned}$ $\quad \begin{aligned} a(2) &= -\frac{3}{4\sqrt{2}} \\ &\approx -0.530 \text{ cm s}^{-2}\end{aligned}$

So, at time $t = 2$ seconds, the particle is about 3.17 cm to the right of the origin, travelling to the right at about 0.879 cm s^{-1}, with decreasing speed $(a(2) \approx -0.530 \text{ cm s}^{-2})$.

page 807 **EXERCISE 19A** Question **4 c**, should read:

4 **c** There is a strong, negative, non-linear correlation between the *number of workers* and *time*.