

ERRATA

Mathematics: Applications and Interpretation HL WORKED SOLUTIONS

First edition - 2020

The following errata were made on or before 10/May/2022

page 523 CHAPTER 12 EXERCISE 12F.3 Question 4 c, should read:

$$4 \quad \text{c} \quad \text{Let } \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \quad \text{If } \mathbf{A}^2 = \mathbf{A}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ \therefore \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Equating corresponding elements:

$$a^2 + bc = a \quad \therefore \quad bc = a(1 - a) \quad \dots (1)$$

$$ab + bd = b \quad \therefore \quad b(a + d - 1) = 0 \quad \dots (2)$$

$$ac + cd = c \quad \therefore \quad c(a + d - 1) = 0 \quad \dots (3)$$

$$bc + d^2 = d \quad \therefore \quad bc = d(1 - d) \quad \dots (4)$$

If $b = c = 0$, then $a = 0$ or 1 and $d = 0$ or 1 .

$$\text{So, } \mathbf{A} \text{ is } \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{ or } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \dots (1)$$

If $b = 0$, $c \neq 0$, then $d = 1 - a$, and $a = 0$ or 1 .

$$\text{So, } \mathbf{A} \text{ is } \begin{pmatrix} 0 & 0 \\ c & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ c & 0 \end{pmatrix}, \quad c \neq 0 \quad \dots (2)$$

If $b \neq 0$, then $d = 1 - a$ and $c = \frac{a(1-a)}{b}$.

$$\text{So, } \mathbf{A} \text{ is } \begin{pmatrix} a & b \\ \frac{a-a^2}{b} & 1-a \end{pmatrix}, \quad b \neq 0.$$

We can combine $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ from (1) with (2) to write the possible matrices $\mathbf{A}$

$$\text{as } \mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ c & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ c & 0 \end{pmatrix} (c \in \mathbb{R}), \quad \begin{pmatrix} a & b \\ \frac{a-a^2}{b} & 1-a \end{pmatrix} (b \neq 0).$$

page 585 CHAPTER 13 ACTIVITY 1 Question 3 b, should read:

- 3 b The highest probability in a is about 0.533 which corresponds to rainy on the first day, then rainy on the second day.
 $\therefore$ the most likely sequence of weather is B.

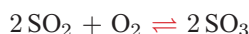
$$\mathbf{x}' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\therefore \mathbf{x}' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} \quad \text{which is of the form } \mathbf{x}' = \mathbf{Ax} + \mathbf{b}.$$

This is the affine transformation composed of an enlargement with scale factor 2 followed by a translation through $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$.

ACTIVITY 2

EQUILIBRIUM REACTIONS



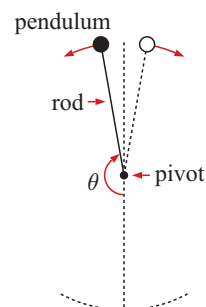
- 5 a From the reaction equation, one unit of SO_2 is used to create one unit of SO_3 and vice versa. So, the total amount of SO_2 and SO_3 in the system must remain constant.
 $\therefore D + T = c$, where c is a constant, which is the equation of a straight line.
 Thus, all trajectories on the diagram must be straight lines of the form $D + T = c$.
- 6 From 5 a and 5 b, the equilibrium point must be the intersection of the lines $D + T = c$ and $T = 4D$.
 So, the initial amount of SO_2 and SO_3 in the system determines the equilibrium point to which the system tends.

- 1 c ii At initial point B, the pendulum is released from rest from a position slightly to the left of the point directly above the pivot.

From B, the pendulum first moves anticlockwise until it comes to rest, then moves clockwise until it comes to rest at the initial position B.

It continues to oscillate in this manner indefinitely.

The maximum angular velocity obtained is greater than in i.



- iii At initial point C, the pendulum is released with large positive angular velocity from the point directly above the pivot.

From C, the pendulum first moves anticlockwise reaching its maximum angular velocity as it passes through its resting position. It continues moving anticlockwise, reaching its minimum angular velocity as it moves through its initial position.

It then continues to move anticlockwise around the pivot, and behaves in this manner indefinitely.

