

ERRATA

Mathematics: Core Topics HL

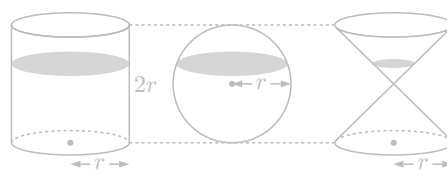
First edition - 2024 fourth reprint

The following erratum was made on 28/Oct/2024

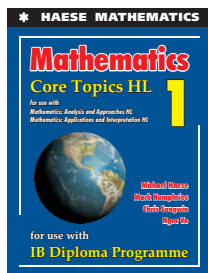
page 146 SECTION 6C INVESTIGATION 2, VOLUME FORMULAE Question 7 b, should read:

- 7** Archimedes used a different idea to find the formula for the volume of a sphere. The following is not exactly what he did, but is very much in the same spirit.

Consider the cylinder, sphere, and double cone alongside.



- a Show that if *any* horizontal slice of the three solids is made, the sum of the areas from the sphere and the double cone equals the area from the cylinder.
- b Explain why the sum of the volumes of the **sphere** and double cone equals the volume of the cylinder.
- c Use the known volume of a cylinder and cone formulae to find the volume of the sphere.



ERRATA

Mathematics: Core Topics HL

First edition - 2021 second reprint

The following errata were made on 14/Oct/2022

page 496 **ANSWERS EXERCISE 5E.3** Question 4 a, should read:

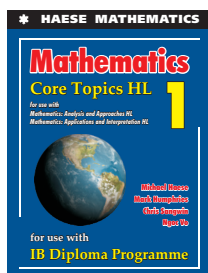
- 4 a Real interest rate = $\frac{1.005}{1.001} \approx 1.003\,996 \approx 0.4\%$
b \$6602.66

page 501 **ANSWERS EXERCISE 8C** Question 8, should read:

- 7 B and D 8 A and C, B and E

page 502 **ANSWERS REVIEW SET 8B** Question 3, should read:

- 2 $\approx 111\text{ cm}^2$ 3 $\approx 103^\circ$ or 257°



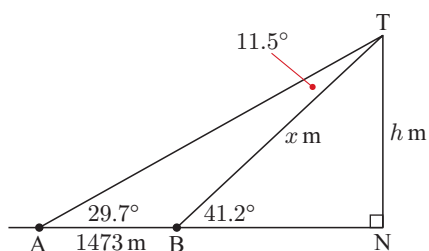
ERRATA

Mathematics: Core Topics HL

First edition - 2020 first reprint

The following erratum was made on 29/Apr/2020

page 222 SECTION 9D Example 8, second line of solution should read:

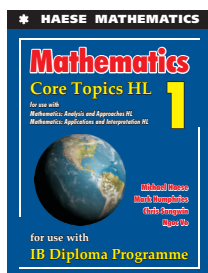


Let BT be x m and NT be h m.

$$\begin{aligned}\widehat{ATB} &= 41.2^\circ - 29.7^\circ \quad \{\text{exterior angle of } \triangle ABT\} \\ &= 11.5^\circ\end{aligned}$$

We find x in $\triangle ABT$ using the sine rule:

$$\begin{aligned}\frac{x}{\sin 29.7^\circ} &= \frac{1473}{\sin 11.5^\circ} \\ \therefore x &= \frac{1473}{\sin 11.5^\circ} \times \sin 29.7^\circ \\ &\approx 3660.62\end{aligned}$$



ERRATA

Mathematics: Core Topics HL

First edition - 2019 initial print

The following errata were made on 30/Mar/2020

page 455 SECTION 17C, blue box should read:

For the general sine function

$$y = a \sin(b(x - c)) + d \quad \text{where } b > 0:$$

affects
amplitude

affects
period

affects
horizontal translation

affects
vertical translation

- the amplitude is $|a|$
- the period is $\frac{2\pi}{b}$
- the principal axis is $y = d$
- $y = a \sin(b(x - c)) + d$ is obtained from $y = \sin x$ by a vertical stretch with scale factor $|a|$ and a horizontal stretch with scale factor $\frac{1}{b}$, a reflection in the x -axis if $a < 0$, and a translation through $\begin{pmatrix} c \\ d \end{pmatrix}$.

page 499 ANSWERS EXERCISE 6D Question 10, should read:

- 10 a $\approx 111\,000 \text{ mm}^2$
 b The external surface area and internal surface area of a container may be different.
 c i $\approx 1\,860\,000 \text{ mm}^3$ ii $\approx 1.86 \text{ L}$ iii $\approx 502\,000 \text{ mm}^3$

page 504 ANSWERS EXERCISE 10B Question 9 b and c, should read:

- 9 a $D(-4, -2, 2)$ b ≈ 14.8 units c $\approx 168 \text{ units}^3$

page 507 ANSWERS EXERCISE 11K Question 5, should read:

- 3 $\frac{10}{13}$ 4 ≈ 0.424 5 ≈ 0.0137 6 $\frac{15}{83}$ 7 $\frac{99}{148}$

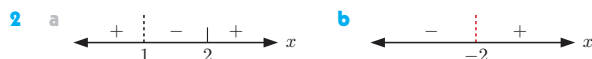
page 516 ANSWERS REVIEW SET 13A Question 8 d, should read:

- 8 a 101.5 b 7.5 c 100.2 d $\sigma \approx 7.59$, $s \approx 7.78$

page 517 ANSWERS REVIEW SET 13B Question 11 b iv, should read:

- 11 b i median ≈ 26 ii IQR ≈ 12
 iii $\bar{x} \approx 26.0$ iv $\sigma \approx 8.31$, $s \approx 8.38$

- 7 a** $y^2 = x$ is a relation but not a function.
 $y = x^2$ is a function (and a relation).
 $y^2 = x$ has a horizontal axis of symmetry (the x -axis).
 $y = x^2$ has a vertical axis of symmetry (the y -axis).
Both $y^2 = x$ and $y = x^2$ pass through $(0, 0)$ and $(1, 1)$.
 $y^2 = x$ is a rotation of $y = x^2$ clockwise through 90°
about the origin or $y^2 = x$ is a reflection of $y = x^2$ in
the line $y = x$.



- 11 a** The inverse function must also be a function and must therefore satisfy the vertical line test, which it can only do if the original function satisfies the horizontal line test.
b i is the only one.
c ii Domain is $\{x \mid x \geq 1\}$ or $\{x \mid x \leq 1\}$
iii Domain is $\{x \mid x \geq 1\}$ or $\{x \mid -2 \leq x \leq 1\}$ or $\{x \mid x \leq -2\}$

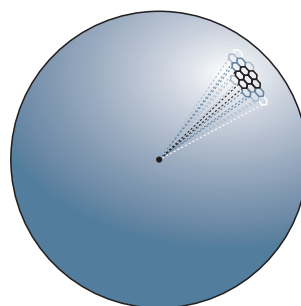
The following errata were made on 04/Dec/2019

17	TRIGONOMETRIC FUNCTIONS	447
A	Periodic behaviour	448
B	The sine and cosine functions	452
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	Review set 17B	479

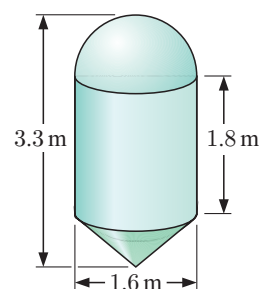
The relevant *Mathematics: Applications and Interpretation* syllabus references are:
SL 2.6, (AI)AHL 2.9, and (AI)AHL 4.13

- 8** Now suppose we approximate a sphere with radius r using a large number n of tapered solids, each with height r and their apex at the centre of the sphere.
Suppose the k th solid has base area A_k .

- a** Explain why $\sum_{k=1}^n \frac{1}{3} A_k r \approx \frac{4}{3} \pi r^3$.
b Hence explain why the surface area of a sphere is given by $A = 4\pi r^2$.

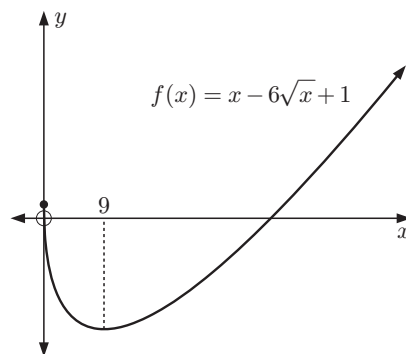


- 11** A feed silo is made out of sheet steel using a hemisphere, a cylinder, and a cone.
a Explain why the height of the cone must be 70 cm.
b Hence find the *slant height* of the conical section.
c Calculate the total amount of steel used.
d Show that the silo can hold about 5.2 cubic metres of grain.
e Write the capacity of the silo in kL.



22 The graph of $f(x) = x - 6\sqrt{x} + 1$ is shown alongside.

- a** State the natural domain of $f(x)$.
- b** Does $f(x)$ have an inverse function? Explain your answer.
- c** Let $g(x) = x - 6\sqrt{x} + 1$, $0 \leq x \leq 9$.
 - i** Find $g^{-1}(x)$, and verify that $(g \circ g^{-1})(0) = 0$.
 - ii** State the domain and range of g and g^{-1} .
- d** Let $h(x) = x - 6\sqrt{x} + 1$, $x \geq 9$.
 - i** Find $h^{-1}(x)$ and verify that $(h \circ h^{-1})(16) = 16$.
 - ii** State the domain and range of h and h^{-1} .
 - iii** Find the value of x such that $g^{-1}(x) = h^{-1}(x)$.



From the **Investigation**, you should have observed that:

- zeros of $y = f(x)$ become vertical asymptotes of $y = \frac{1}{f(x)}$
- vertical asymptotes of $y = f(x)$ become zeros of $y = \frac{1}{f(x)}$
- local maxima of $y = f(x)$, $f(x) \neq 0$ correspond to local minima of $y = \frac{1}{f(x)}$
- local minima of $y = f(x)$, $f(x) \neq 0$ correspond to local maxima of $y = \frac{1}{f(x)}$
- when $f(x) > 0$, $\frac{1}{f(x)} > 0$ and when $f(x) < 0$, $\frac{1}{f(x)} < 0$
- when $f(x) \rightarrow 0$, $\frac{1}{f(x)} \rightarrow \pm\infty$ and when $f(x) \rightarrow \pm\infty$, $\frac{1}{f(x)} \rightarrow 0$.

Questions for this section apply to all *Mathematics: HL* students, so they should not be coloured green.

- 16 Hint:** Write out the terms of the sequence in order, then write out the terms in reverse order directly underneath. Add the terms vertically.

- 8 a** 4 units **b** $k = -3 \pm \sqrt{6}$ **c** ≈ 142 units²

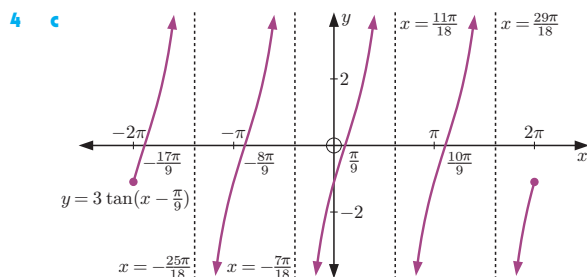
- 6 a** $\approx 21.4^\circ$ **b** ≈ 3.54 units²

- 6 a** i $k < \frac{9}{4}$ ii $k = \frac{9}{4}$ iii $k > \frac{9}{4}$
b i $k < 4$, $k \neq 0$ ii $k = 4$ iii $k > 4$
c i $k > -\frac{4}{3}$, $k \neq -1$ ii $k = -\frac{4}{3}$ iii $k < -\frac{4}{3}$

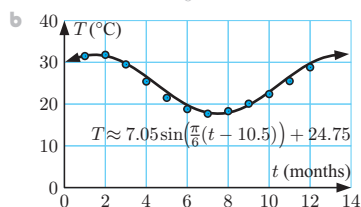
- 6 a** Domain is $\{x \mid x \neq -\frac{d}{c}\}$
b vertical asymptote is $x = -\frac{d}{c}$
c x -intercept is $-\frac{b}{a}$, $a \neq 0$, y -intercept is $\frac{b}{d}$, $d \neq 0$
d $\frac{ax+b}{cx+d} = \frac{\frac{a}{c}(cx+d) - \frac{ad}{c} + b}{cx+d}$ and so on
 As $|x| \rightarrow \infty$, $\frac{b - \frac{ad}{c}}{cx+d} \rightarrow 0$.
 $\therefore$ the horizontal asymptote is $y = \frac{a}{c}$.

- 11 a** The graph is stretched vertically with scale factor $|a|$, and reflected in the x -axis. It is then translated h units horizontally and k units vertically.
b The function has shape  after it is reflected in the x -axis.
 The function has vertex (h, k) , and y -intercept $ah^2 + k$.

Questions for this section apply to all *Mathematics: HL* students, so they should not be coloured green.



- 14 a** $a \approx 7.05$, $b \approx \frac{\pi}{6}$, $c \approx 10.5$, $d \approx 24.75$



- c** Using technology, $T \approx 7.20 \sin(0.488t + 1.08) + 24.7$.
 The model fits reasonably well but not perfectly.

The following errata were made on 01/Nov/2019

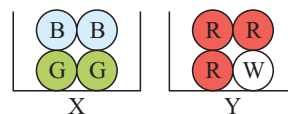
page 69 REVIEW SET 3A Question 13, should read:

- 13** At a particular moment, Earth is 4.3×10^9 km from Neptune and 1.5×10^9 km from Saturn.
How many times further away is Neptune than Saturn?

page 268 SECTION 11G INVESTIGATION 4, INDEPENDENT EVENTS Question 2 a, should read:

- 2** Consider randomly selecting a ball from each of the boxes alongside.

- a** Does the outcome of the draw from either box affect the outcome of the other?



page 351 SECTION 13J INVESTIGATION 3, TRANSFORMING DATA Question 3 b, should read:

- 3** **a** Suppose we multiply each value in the original data set by 4. Calculate the mean and population standard deviation for the new data set.
b What do you expect to happen to the mean and standard deviation if each value in a data set is multiplied by $a > 0$?

page 352 SECTION 13J INVESTIGATION 3, TRANSFORMING DATA Question 4, should read:

- 4** Suppose a data set $\{x_i\}$ has mean μ and standard deviation σ . Write down the mean and standard deviation for the data set:
a $\{ax_i\}$, $a > 0$ **b** $\{x_i + k\}$ **c** $\{ax_i + k\}$, $a > 0$

page 457 EXERCISE 17C Question 14, should read:

- 14** Consider the general sine function $y = a \sin(b(x - c)) + d$, with default values $a, b = 1$, $c, d = 0$. State which of the variables a, b, c , and d can be changed to produce a change in:
a the x -intercepts of the function **b** the y -intercept of the function
c the range of the function.

page 487 ANSWERS EXERCISE 2E Question 4 c, should read:

- 4** **a** $\{x \mid x \geq 8\}$ **b** $\{x \mid -1 \leq x < 4\}$
c $\{x \in \mathbb{Z} \mid -3 < x < 4\}$ or $\{x \in \mathbb{Z} \mid -2 \leq x \leq 3\}$
d $\{x \in \mathbb{N} \mid x \leq 4\} \cup \{x \in \mathbb{N} \mid x = 6\}$

page 492 ANSWERS REVIEW SET 3A Question 13, should read:

- 12** 313 sheets **13** ≈ 2.87 times

page 496 ANSWERS EXERCISE 5F Question 8, should read:

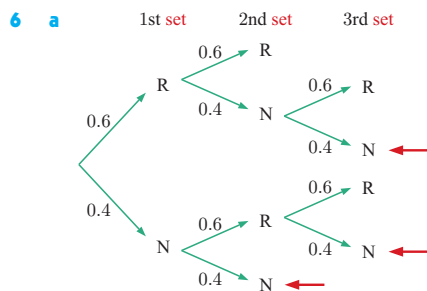
8 $S_n = \sum_{k=1}^n (2k - 1) = n^2$

page 497 ANSWERS REVIEW SET 5A Question 14 a, should read:

- 13** \$4800 **14** **a** €6622.88 **b** €13 313.28

page 507 ANSWERS EXERCISE 11I Question 10 a, should read:

- 10** **a** $\frac{31}{52}$ **b** $\frac{70}{163}$ **11** $\frac{7}{15}$



7 d i ≈ 25.6 cm ii ≈ 18 people

14 a *b, c, d* **b** *c, d* **c** *a, d*

The following errata were made on 23/Oct/2019

- 9** Each year, a school manages to use only 90% as much paper as the previous year. In the year 2010, they used 700 000 sheets of paper.
- a** Find how much paper the school used in the years 2011 and 2012.
 - b** How much paper did the school use in total in the decade from 2008 to 2017?

- 9** Consider the frequency table alongside:
- a** Find the values of p and m .
 - b** Hence find the mode, median, and range of the data.
 - c** Given that $\sum xf = 254$, write the mean $\bar{x}$ as a fraction.

Score (x)	Frequency (f)	Cumulative frequency
6	2	2
7	4	m
8	7	13
9	p	25
10	5	30

- | | | |
|---|--|---|
| <p>a The graph cuts the x-axis twice if $\Delta > 0$.
 $\therefore 36 - 4k > 0$
 $\therefore 4k < 36$
 $\therefore k < 9$</p> | <p>b The graph touches the x-axis if $\Delta = 0$.
 $\therefore 36 - 4k = 0$
 $\therefore k = 9$</p> | <p>c The graph does not cut the x-axis if $\Delta < 0$.
 $\therefore 36 - 4k < 0$
 $\therefore 4k > 36$
 $\therefore k > 9$</p> |
|---|--|---|

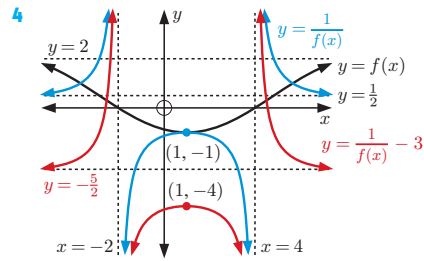
- 9** b_1, c_1, b_2 , and c_2 are real, non-zero numbers such that $b_1b_2 = 2(c_1 + c_2)$. Show that at least one of the quadratics $y = x^2 + b_1x + c_1$, and $y = x^2 + b_2x + c_2$ **meets the x -axis**.

3 a $\theta \approx 1.25$ **b** $\theta \approx -0.983$ **c** $\theta \approx 2.76$

page 523 **ANSWERS EXERCISE 15C** Question **1 c**, should read:

1 c Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y = 2, 3, 5, 7, \text{ or } 9\}$

page 536 **ANSWERS EXERCISE 16E** Question **4**, should have correct horizontal asymptote for $y = \frac{1}{f(x)} - 3$:



page 544 **ANSWERS REVIEW SET 17A** Question **14 e**, should use correct variable name:

14 e Using technology,
 $T \approx 36.5 \sin(0.009\,01n - 0.0903) - 43.2$.
Our model fits the data well.