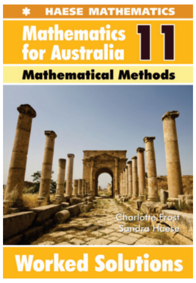


ERRATA

MATHEMATICS FOR AUSTRALIA 11

Mathematical Methods

Worked Solutions

First edition - 2017

The following erratum was made on 05/May/2020

page 509 **EXERCISE 15F** Question **7 b**, sign diagram should be:

7 **b** Area of rectangle $A = 2x \times l$

$$= 2x \times (200 - \pi x)$$

$$\therefore \quad A = 400x - 2\pi x^2$$

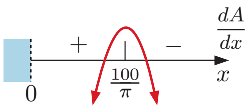
$$\frac{dA}{dx} = 400 - 4\pi x$$

$$\frac{dA}{dx} = 0 \quad \text{when} \quad 400 - 4\pi x = 0$$

$$\therefore \quad 4\pi x = 400$$

$$\therefore \quad x = \frac{100}{\pi}$$

$\frac{dA}{dx}$ has sign diagram



The following errata were made on 28/Apr/2020

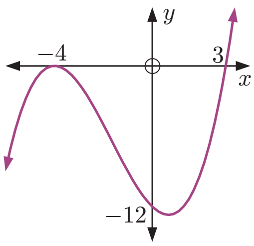
page 134 **EXERCISE 4E** Question **3 c**, should read:

3 **c** The graph touches the x-axis at -4, indicating a squared factor $(x+4)^2$.
The other x-intercept is 3,
so $y = a(x+4)^2(x-3)$.
But when $x = 0$, $y = -12$

$$\therefore \quad a(4)^2(-3) = -12$$

$$\therefore \quad a = \tfrac{1}{4}$$

So, $y = \frac{1}{4}(x+4)^2(x-3)$

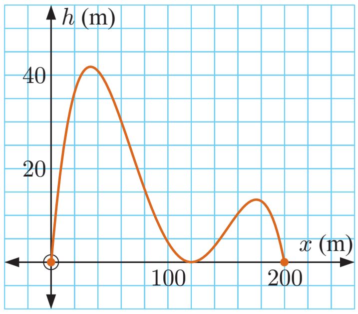


page 144 **EXERCISE 4F** Question **10 a** and **b**, should read:

10 **a** The graph touches the x-axis at 120, and cuts the x-axis at 0 and 200.

$$\therefore \quad h(x) = \frac{-x(x-120)^2(x-200)}{k}, \quad k \neq 0$$

So, $a = 120$ and $b = 200$.



b When $x = 100$, $h(x) = 4$

$$\therefore \quad 4 = \frac{(-100)(100-120)^2(100-200)}{k}$$

$$= \frac{4\,000\,000}{k}$$

$$\therefore \quad 4k = 4\,000\,000$$

$$\therefore \quad k = 1\,000\,000$$

page 240 **EXERCISE 7G** Question **4 a**, should read:

4 **a** $\cos 2\theta \cos\theta + \sin 2\theta \sin\theta$
$= \cos(2\theta - \theta)$
$= \cos\theta$

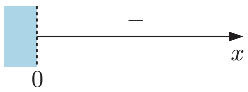
b $\sin 2A \cos A + \cos 2A \sin A$
$= \sin(2A + A)$
$= \sin 3A$

page 488 **EXERCISE 15C** Question **3 e**, sign diagram should be:

3 **e** $f(x) = \dfrac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}}$

$\therefore \quad f'(x) = -x^{-\frac{3}{2}} = -\dfrac{1}{x\sqrt{x}}$

which has sign diagram

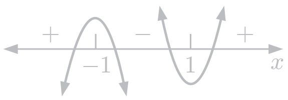


So, $f(x)$ is only defined for $x > 0$.
$f(x)$ is never increasing, but is decreasing for $x > 0$.

page 492 **EXERCISE 15D** Question **5 d**, sign diagram should be:

5 **c** $f(x) = x^3 - 3x + 2$
$\therefore \quad f'(x) = 3x^2 - 3$
$= 3(x^2 - 1)$
$= 3(x+1)(x-1)$

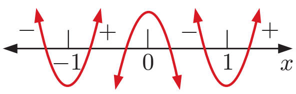
which has sign diagram



Now $f(-1) = 4$, $f(1) = 0$, so there is a local maximum at $(-1, 4)$, and a local minimum at $(1, 0)$.

d $f(x) = x^4 - 2x^2$
$\therefore \quad f'(x) = 4x^3 - 4x$
$= 4x(x^2 - 1)$
$= 4x(x+1)(x-1)$

which has sign diagram



Now $f(-1) = -1$, $f(1) = -1$, $f(0) = 0$, so there are local minima at $(-1, -1)$ and $(1, -1)$, and a local maximum at $(0, 0)$.

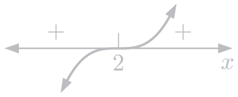
page 493 **EXERCISE 15D** Question **5 f**, sign diagram should be:

5 **e** $f(x) = x^3 - 6x^2 + 12x + 1$

$\therefore \quad f'(x) = 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)^2$

which has sign diagram 

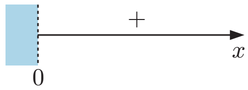
Now $f(2) = 9$, so there is a stationary inflection at $(2, 9)$.

f $f(x) = \sqrt{x} + 2$

$= x^{\frac{1}{2}} + 2$

$\therefore \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}} \neq 0$

which has sign diagram 

$\therefore$ there are no stationary points.

page 497 **EXERCISE 15D** Question **10 d**, sign diagram should be:

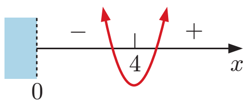
10 **d** Let $f(x) = x - 4\sqrt{x} = x - 4x^{\frac{1}{2}}$, for $0 \leqslant x \leqslant 5$

$\therefore \quad f'(x) = 1 - 2x^{-\frac{1}{2}}$

$= 1 - \frac{2}{\sqrt{x}}$

$= \frac{\sqrt{x} - 2}{\sqrt{x}}$

which is 0 when $x = 4$

The sign diagram of $f'(x)$ is: 

$\therefore$ there is a local minimum at $x = 4$.

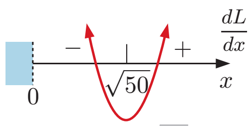
page 505 **EXERCISE 15F** Question **2 b**, sign diagram should be:

2 **b** $\frac{dL}{dx} = 2 - 100x^{-2} = 2 - \frac{100}{x^2}$

which is 0 when $\frac{100}{x^2} = 2$

$\therefore \quad x^2 = 50$

$\therefore \quad x = \sqrt{50} \quad \{x > 0\}$

So, $\frac{dL}{dx}$ has sign diagram 

So, the length is a minimum when $x = \sqrt{50}$.

page 507 **EXERCISE 15F** Question **4 c**, sign diagram should be:

4 **c** $A'(x) = 8x - 600x^{-2}$

$= 8x - \frac{600}{x^2}$

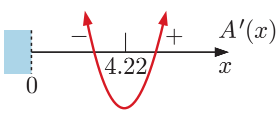
$A'(x) = 0$ when $8x - \frac{600}{x^2} = 0$

$\therefore \quad 8x = \frac{600}{x^2}$

$\therefore \quad 8x^3 = 600$

$\therefore \quad x^3 = 75$

$\therefore \quad x = \sqrt[3]{75} \approx 4.22$

$A'(x)$ has sign diagram 

page 508 **EXERCISE 15F** Question **5 c**, sign diagram should be:

5 **c** $\dfrac{dA}{dr} = 4\pi r - 2000r^{-2}$

$= 4\pi r - \dfrac{2000}{r^2}$

$\dfrac{dA}{dr} = 0$ when $4\pi r - \dfrac{2000}{r^2} = 0$

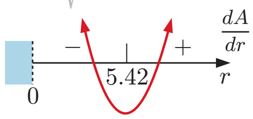
$\therefore\ 4\pi r = \dfrac{2000}{r^2}$

$\therefore\ 4\pi r^3 = 2000$

$\therefore\ r^3 = \dfrac{500}{\pi}$

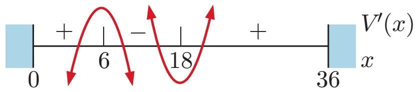
$\therefore\ r = \sqrt[3]{\dfrac{500}{\pi}} \approx 5.42$

$\dfrac{dA}{dr}$ has sign diagram



page 509 **EXERCISE 15F** Question **6 b**, sign diagram should be:

6 **b** $V'(x)$ has sign diagram



page 520 **EXERCISE 15F** Question **12 c**, sign diagram should be:

12 **c** $C(x) = 2x^2 + \dfrac{8}{x} = 2x^2 + 8x^{-1}$

$\therefore\ C'(x) = 4x - 8x^{-2} = 4x - \dfrac{8}{x^2}$

$C'(x) = 0$ when $4x - \dfrac{8}{x^2} = 0$

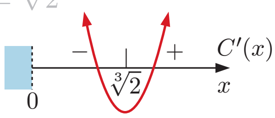
$\therefore\ 4x = \dfrac{8}{x^2}$

$\therefore\ 4x^3 = 8$

$\therefore\ x^3 = 2$

$\therefore\ x = \sqrt[3]{2}$

$C'(x)$ has sign diagram



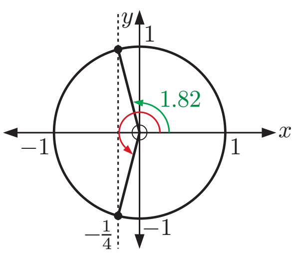
The following errata were made on 31/Oct/2019

pages 166-168 **EXERCISE 5D.2** Question **2**, diagrams should distinguish calculator answers (using green):

2 **a** $\cos\theta = -\frac{1}{4}$

Using technology,

$\cos^{-1}(-\frac{1}{4}) \approx 1.82$

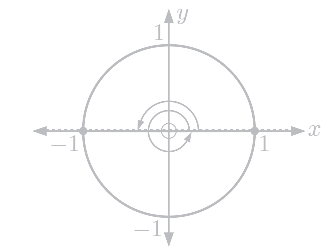


$\therefore\ \theta \approx 1.82$ or $2\pi - 1.82$

$\therefore\ \theta \approx 1.82$ or 4.46

b $\sin\theta = 0$

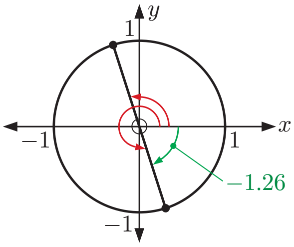
$\therefore\ \sin^{-1}(0) = 0$



$\therefore\ \theta = 0$ or $\pi - 0$ or 2π

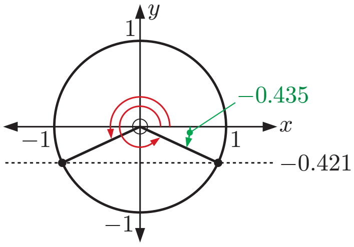
$\therefore\ \theta = 0, \pi,$ or 2π

c $\tan\theta = -3.1$
Using technology,
$\tan^{-1}(-3.1) \approx -1.26$



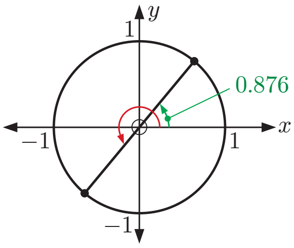
But $0 \leqslant \theta \leqslant 2\pi$
$\therefore \quad \theta \approx \pi - 1.26 \quad \text{or} \quad 2\pi - 1.26$
$\therefore \quad \theta \approx 1.88 \quad \text{or} \quad 5.02$

d $\sin\theta = -0.421$
Using technology,
$\sin^{-1}(-0.421) \approx -0.435$



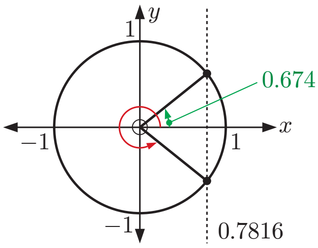
But $0 \leqslant \theta \leqslant 2\pi$
$\therefore \quad \theta \approx \pi + 0.435 \quad \text{or} \quad 2\pi - 0.435$
$\therefore \quad \theta \approx 3.58 \quad \text{or} \quad 5.85$

e $\tan\theta = 1.2$
Using technology,
$\tan^{-1}(1.2) \approx 0.876$



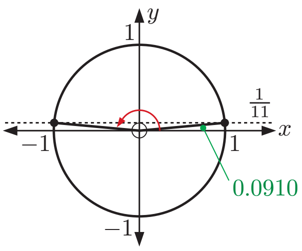
$\therefore \quad \theta \approx 0.876 \quad \text{or} \quad \pi + 0.876$
$\therefore \quad \theta \approx 0.876 \quad \text{or} \quad 4.02$

f $\cos\theta = 0.7816$
Using technology,
$\cos^{-1}(0.7816) \approx 0.674$



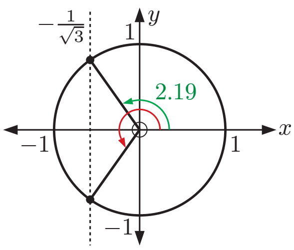
$\therefore \quad \theta \approx 0.674 \quad \text{or} \quad 2\pi - 0.674$
$\therefore \quad \theta \approx 0.674 \quad \text{or} \quad 5.61$

g $\sin\theta = \frac{1}{11}$
Using technology,
$\sin^{-1}(\frac{1}{11}) \approx 0.0910$



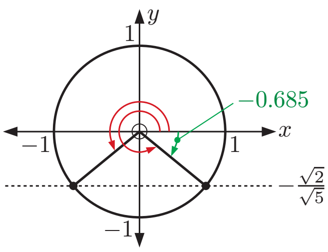
$\therefore \quad \theta \approx 0.0910 \quad \text{or} \quad \pi - 0.0910$
$\therefore \quad \theta \approx 0.0910 \quad \text{or} \quad 3.05$

h $\cos\theta = -\frac{1}{\sqrt{3}}$
Using technology,
$\cos^{-1}(-\frac{1}{\sqrt{3}}) \approx 2.19$



$\therefore \quad \theta \approx 2.19 \quad \text{or} \quad 2\pi - 2.19$
$\therefore \quad \theta \approx 2.19 \quad \text{or} \quad 4.10$

i $\sin\theta = -\frac{\sqrt{2}}{\sqrt{5}}$
Using technology,
$\sin^{-1}(-\frac{\sqrt{2}}{\sqrt{5}}) \approx -0.685$

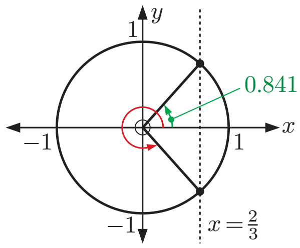


But $0 \leqslant \theta \leqslant 2\pi$
$\therefore \quad \theta \approx \pi + 0.685 \quad \text{or} \quad 2\pi - 0.685$
$\therefore \quad \theta \approx 3.83 \quad \text{or} \quad 5.60$

page 182 **REVIEW SET 5A** Question **16**, diagrams should distinguish calculator answers (using green):

16 **a** $\cos\theta = \frac{2}{3}$

Using technology, $\cos^{-1}(\frac{2}{3}) \approx 0.841$

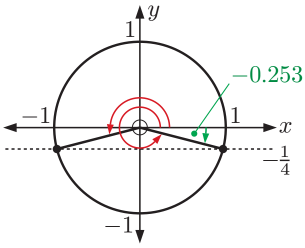


$\therefore\ \theta \approx 0.841$ or $2\pi - 0.841$

$\therefore\ \theta \approx 0.841$ or 5.44

b $\sin\theta = -\frac{1}{4}$

Using technology, $\sin^{-1}(-\frac{1}{4}) \approx -0.253$



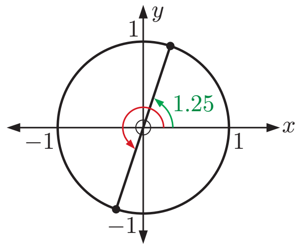
But $0 \leqslant \theta \leqslant 2\pi$

$\therefore\ \theta \approx \pi + 0.253$ or $2\pi + (-0.253)$

$\therefore\ \theta \approx 3.39$ or 6.03

c $\tan\theta = 3$

Using technology, $\tan^{-1}(3) \approx 1.25$



$\therefore\ \theta \approx 1.25$ or $\pi + 1.25$

$\therefore\ \theta \approx 1.25$ or 4.39

The following errata were made on 23/Oct/2019

page 68 **EXERCISE 2H** Question **8 b**, should show shape of quadratic:

8 **b** Rectangle ABCD has area $A = xy$

$= x(6 - \frac{3}{4}x)$

$= -\frac{3}{4}x^2 + 6x$

which is a quadratic with $a < 0$, so its shape is 

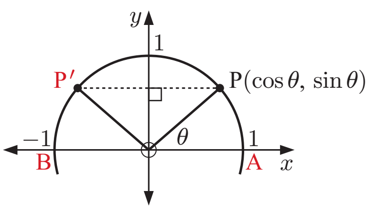
The area is maximised when $x = \dfrac{-b}{2a} = \dfrac{-6}{2(-\frac{3}{4})} = 4$

and when $x = 4$, $y = 6 - \frac{3}{4}(4) = 3$

So, the dimensions of rectangle ABCD of maximum area are 4 cm × 3 cm.

page 160 **EXERCISE 5C** Question **7 c**, diagram should show labels:

7 **c**



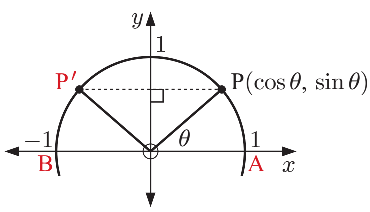
The diagram shows P reflected in the y-axis to P′, so $P'\widehat{O}B = P\widehat{O}A = \theta$, and P′ has coordinates $(-\cos\theta, \sin\theta)$.

But $A\widehat{O}P' = 180° - \theta$ $\{A\widehat{O}P' + P'\widehat{O}B = 180°\}$, so P′ has coordinates $(\cos(180° - \theta), \sin(180° - \theta))$.

$\therefore\ \sin(180° - \theta) = \sin\theta$ {equating y-coordinates of P′}

page 161 **EXERCISE 5C** Question **8 c**, diagram should show labels:

8 **c**



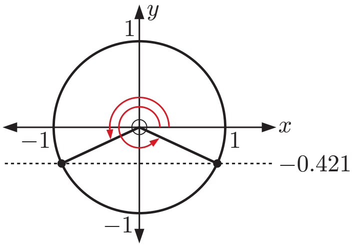
The diagram shows P reflected in the y-axis to P′, so $P'\widehat{O}B = P\widehat{O}A = \theta$, and P′ has coordinates $(-\cos\theta, \sin\theta)$.

But $A\widehat{O}P' = 180° - \theta$ $\{A\widehat{O}P' + P'\widehat{O}B = 180°\}$, so P′ has coordinates $(\cos(180° - \theta), \sin(180° - \theta))$.

$\therefore\ \cos(180° - \theta) = -\cos\theta$ {equating x-coordinates of P′}

page 167 **EXERCISE 5D.2** Question **2 d**, should have correct diagram:

2 **d** $\sin\theta = -0.421$
Using technology,
$\sin^{-1}(-0.421) \approx -0.435$



But $0 \leqslant \theta \leqslant 2\pi$
$\therefore\ \theta \approx \pi + 0.435$ or $2\pi - 0.435$
$\therefore\ \theta \approx 3.58$ or 5.85

The following errata were made on 02/Sep/2019

page 318 **EXERCISE 10B** Question **14 a**, should read:

14 **a** *Month 1*: 5 cars; *Month 2*: $5 + 13 = 18$ cars
Month 3: $18 + 13 = 31$ cars; *Month 4*: $31 + 13 = 44$ cars
Month 5: $44 + 13 = 57$ cars; *Month 6*: $57 + 13 = 70$ cars

page 340 **EXERCISE 10F.2** Question **7**, should read:

7 Let the terms of the geometric series be $t_1,\ t_1r,\ t_1r^2,\ ....$

Then $t_1r = \frac{8}{5}$ and $\dfrac{t_1}{1-r} = 10$

$\therefore\ t_1 = \dfrac{8}{5r}$ .... (1) $\qquad \therefore\ t_1 = 10 - 10r$ .... (2)

Equating (1) and (2), $\dfrac{8}{5r} = 10 - 10r$

$$\therefore\ 8 = 50r - 50r^2$$
$$\therefore\ 50r^2 - 50r + 8 = 0$$
$$\therefore\ 2(25r^2 - 25r + 4) = 0$$
$$\therefore\ 2(5r-1)(5r-4) = 0$$
$$\therefore\ r = \tfrac{1}{5} \text{ or } \tfrac{4}{5}$$

Using (2), if $r = \frac{1}{5}$, $t_1 = 10 - 10(\frac{1}{5}) = 8$
if $r = \frac{4}{5}$, $t_1 = 10 - 10(\frac{4}{5}) = 2$

$\therefore$ either $t_1 = 8,\ r = \frac{1}{5}$ or $t_1 = 2,\ r = \frac{4}{5}$.