

ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) (3rd edition)

Third edition - 2018 fourth reprint

The following erratum was made on 25/May/2020

page 933 INVESTIGATION 2 **3**, should read:

3 Use the midpoint rule to estimate:

a $\int_0^2 \sin(x^2) dx$

b the area enclosed between $y = \sin(x^2)$, the x -axis, and the vertical lines $x = 0$ and $x = 2$.

The following erratum was made on 04/Mar/2020

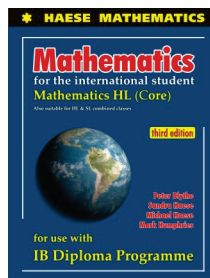
page 933 ANSWERS EXERCISE 20B **18**, should read:

- 18 b i** 0 m per radian **ii** 1 m per radian
iii ≈ 1.11 m per radian

The following erratum was made on 21/Feb/2020

page 955 ANSWERS EXERCISE 27A **79**, should read:

79 $x = -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12}$



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) (3rd edition)

Third edition - 2017 third reprint

The following erratum was made on 25/Jul/2017

page 623 REVIEW SET 20A 10 b, should read:

- 10 b** Show that if $\theta = \widehat{APM} = \widehat{BPM}$, then the length of cable is given by

$$L(\theta) = 2 \csc \theta + 3 - \cot \theta \text{ km.}$$

The following erratum was made on 30/Jan/2017

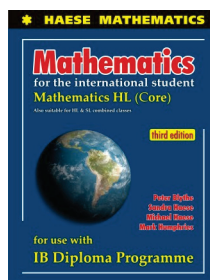
page 863 EXERCISE 27B 98, should read:

- 98** Suppose 3 , $3 \log_y x$, $3 \log_z y$, and $7 \log_x z$ are consecutive terms of an arithmetic sequence.
- a** Prove that $x^{18} = y^{21} = z^{28}$.
- b** Find x in terms of: **i** y **ii** z .

The following erratum was made on 19/Dec/2016

page 872 ANSWERS EXERCISE 2J 12 c, should have parts **ii** and **iii** correctly labelled and include domains on both sides:

- 12 b** **i** is the only one
- c** **ii** Domain = $\{x \mid x \leq 1\}$ or Domain = $\{x \mid x \geq 1\}$
iii Domain = $\{x \mid x \geq 1\}$ or Domain = $\{x \mid x \leq -2\}$



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) (3rd edition)

Third edition - 2015 second reprint

The following erratum was made on 28/Feb/2018

page 721 **SECTION 23C, VARIANCE AND STANDARD DEVIATION** should lose the second paragraph and read:

VARIANCE AND STANDARD DEVIATION

The problem with using the range and the IQR as measures of spread or dispersion of scores is that both of them only use two values in their calculation. Some data sets can therefore have their spread characteristics hidden when the range or IQR are quoted. We therefore turn to the **variance** and **standard deviation** of a data set.

Consider a data set of n values: $x_1, x_2, x_3, x_4, \dots, x_n$, with mean $\bar{x}$.

$x_i - \bar{x}$ measures how far x_i deviates from the mean, so one might suspect that the mean of the deviations $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})$ would give a good measure of spread. However, this value turns out to always be zero.

Instead, we define the **variance of n data values to be** $\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$.

Notice in this formula that:

- $(x_i - \bar{x})^2$ is also a measure of how far x_i deviates from $\bar{x}$. However, the square ensures that each term in the sum is positive, which is why the sum turns out not to be zero.
- If $\sum_{i=1}^n (x_i - \bar{x})^2$ is small, it will indicate that most of the data values are close to $\bar{x}$.
- Dividing by n gives an indication of how far, on average, the data is from the mean.

For a data set of n values, $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$ is called the **standard deviation**.

The square root in the standard deviation is used to correct the units. For example, if x_i is the weight of a student in kg, σ^2 would be in kg^2 . For this reason the standard deviation is more frequently quoted than the variance.

For the purpose of this course, we assume the formulae for variance and standard deviation of a whole population are the same as those for a sample.

The following errata were made on 02/Nov/2016

page 72 **EXAMPLE 15**, last line of solution should read:

$$1 \leq x < 2 \quad \text{or} \quad x \geq 5.5.$$

page 660 **EXERCISE 21H**, advice about integration by parts should read:

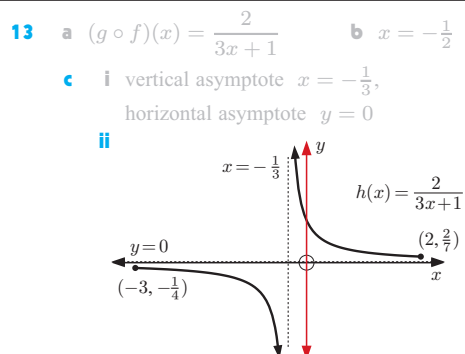
When using 'integration by parts' the function u should be easy to differentiate and the function v' should be easy to integrate.



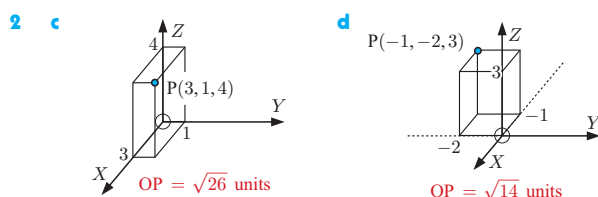
page 869 **ANSWERS EXERCISE 2D 7 b**, should read:

7 b $(g \circ f)(x) = 2x + 5$, $x \neq -2$, Domain = $\{x \mid x \neq -2\}$

page 874 **ANSWERS REVIEW SET 2B 13 c ii**, should have y axis:



page 911 **ANSWERS EXERCISE 14G 2 c** and **d**, should state length of OP:



page 916 **ANSWERS EXERCISE 15H.1 6 b**, should read:

- 6 b** If $k \neq 4$, the system is inconsistent and so has no solutions.
The lines are parallel.
If $k = 4$, the system has infinitely many solutions of the form $x = t$, $y = 3t - 2$, $t \in \mathbb{R}$. The lines are coincident.

page 921 **ANSWERS EXERCISE 16F.2 4**, Hint is for **4 b ii** not **4 a ii**:

- 4 b ii** **Hint:** The LHS is a geometric series.

The following errata were made on 10/Jun/2016

page 47 **EXERCISE 1H 11**, should read:

- 11** b_1 , c_1 , b_2 , and c_2 are real, **non-zero** numbers such that $b_1 b_2 = 2(c_1 + c_2)$. Show that at least one of the equations $x^2 + b_1 x + c_1 = 0$, $x^2 + b_2 x + c_2 = 0$ has two real roots.

page 84 **SELF-INVERSE FUNCTIONS first dot point**, should read:

For example:

- The function $f(x) = x$ is the **identity** function, and is also a self-inverse function.

page 86 **EXERCISE 2J 14 b iv**, should read:

- 14 a** Explain why $f(x) = x^2 - 4x + 3$ is a function but does not have an inverse function.
- b i** Explain why $g(x) = x^2 - 4x + 3$ where $x \geq 2$ has an inverse function.
- ii** Show that the inverse function is $g^{-1}(x) = 2 + \sqrt{1+x}$.
- iii** State the domain and range of:
A g **B** g^{-1} .
- iv** Show that $(g \circ g^{-1})(x) = (g^{-1} \circ g)(x)$, the **identity** function.

page 193 **THE REMAINDER THEOREM first line**, should clarify:

Consider the **real** cubic polynomial $P(x) = x^3 + 5x^2 - 11x + 3$.

page 210 **REVIEW SET 6B 1**, should declare x and y are elements of the set of integers towards the end of the question:

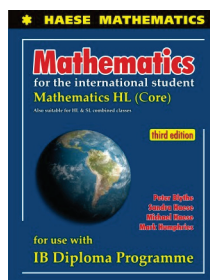
1 If $z = \left(\frac{5}{2-i} - 3 - 2i\right)^3$, express z in the form $z = x + yi$, $x, y \in \mathbb{Z}$.

page 498 **first line after proof box**, should quote property of cis correctly:

We also observe that $\text{cis } (-n\theta) = \text{cis } (0 - n\theta) = \frac{\text{cis } 0}{\text{cis } n\theta}$ {as $\frac{\text{cis } \theta}{\text{cis } \phi} = \text{cis } (\theta - \phi)$ }

page 503 **EXERCISE 16F.2 1**, should read:

1 In **Example 23** we showed that the cube roots of 1 are 1, w , w^2 where $w = \text{cis } \left(\frac{2\pi}{3}\right)$.



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) (3rd edition)

Third edition - 2013 first reprint

The following errata were made on 21/Oct/2015

page 952 **ANSWERS REVIEW SET 25C 10 a ii**, should read:

- 10 a** i $\mu = 1.28, \sigma = 1.13$ **b** ≈ 0.366
 ii $E(Y) = 1.14, \sigma_Y = 0.566$

The following errata were made on or before 12/Jan/2015

page 26 **EXAMPLE 10 Answers parts a and b**, should read:

- a** For two distinct real roots, $\Delta > 0 \quad \therefore k < -9 \text{ or } k > -1, k \neq 0.$
b For two real roots, $\Delta \geq 0 \quad \therefore k \leq -9 \text{ or } k \geq -1, k \neq 0.$

page 43 **PART G Problem solving with quadratics**, second dot point should read:

Any answer we obtain must be checked to see if it is reasonable. For example:

- if we are finding a length then it must be positive and we reject any negative solutions
- if we are finding 'how many people are present' then clearly the answer must be a **positive** integer.

page 130 **Proof at bottom of the page**, should read:

• $\log_c (AB)$ $= \log_c (c^{\log_c A} \times c^{\log_c B})$ $= \log_c (c^{\log_c A + \log_c B})$ $= \log_c A + \log_c B$	• $\log_c \left(\frac{A}{B} \right)$ $= \log_c \left(\frac{c^{\log_c A}}{c^{\log_c B}} \right)$ $= \log_c (c^{\log_c A - \log_c B})$ $= \log_c A - \log_c B$
• $\log_c (A^n)$ $= \log_c ((c^{\log_c A})^n)$ $= \log_c (c^{n \log_c A})$ $= n \log_c A$	

page 188 **DIVISION BY QUADRATICS**, blue box at top of the page should read:

If $P(x)$ is divided by $ax^2 + bx + c$ then

$$\frac{P(x)}{ax^2 + bx + c} = Q(x) + \frac{ex + f}{ax^2 + bx + c} \quad \text{where } ax^2 + bx + c \text{ is the divisor,}$$

$$Q(x) \text{ is the quotient,}$$

$$\text{and } ex + f \text{ is the remainder.}$$

The remainder will be linear if $e \neq 0$, and constant if $e = 0$.

In fact, $n^2 + n + 41$ is prime for all positive integers n from 1 to 39. However, checking $n = 40$ gives a counter-example to the statement, since $40^2 + 40 + 41 = 41 \times 41$.

PERIODICITY OF TRIGONOMETRIC RATIOS

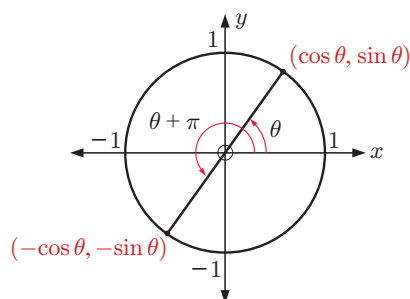
Since there are 2π radians in a full revolution, if we add any integer multiple of 2π to θ (in radians) then the position of P on the unit circle is unchanged.

For θ in radians and $k \in \mathbb{Z}$,

$$\cos(\theta + 2k\pi) = \cos \theta \text{ and } \sin(\theta + 2k\pi) = \sin \theta.$$

We notice that for any point $(\cos \theta, \sin \theta)$ on the unit circle, the point directly opposite is $(-\cos \theta, -\sin \theta)$.

$$\begin{aligned} \therefore \cos(\theta + \pi) &= -\cos \theta \\ \sin(\theta + \pi) &= -\sin \theta \\ \text{and } \tan(\theta + \pi) &= \tan \theta \end{aligned}$$



$$\text{For } \theta \text{ in radians and } k \in \mathbb{Z}, \quad \tan(\theta + k\pi) = \tan \theta.$$

REPRESENTING CONJUGATES

If $z = x + iy$, then $z^* = x - iy$.

This means that if $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix}$ represents z , then $\overrightarrow{OP'} = \begin{pmatrix} x \\ -y \end{pmatrix}$ represents z^* .

For example:

$\overrightarrow{OP}$ represents $2 + 4i$ and

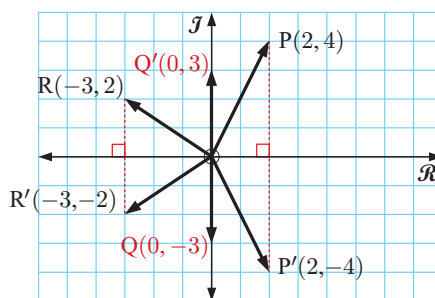
$\overrightarrow{OP'}$ represents $2 - 4i$.

$\overrightarrow{OQ}$ represents $-3i$ and

$\overrightarrow{OQ'}$ represents $3i$.

$\overrightarrow{OR}$ represents $-3 + 2i$ and

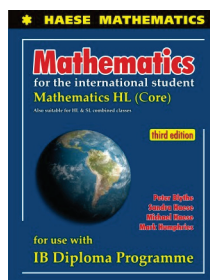
$\overrightarrow{OR'}$ represents $-3 - 2i$.



We often judge performances by rates. For example:

- Sir Donald Bradman's average batting rate at Test cricket level was 99.94 runs per innings.
- Michael Jordan's average basketball scoring rate was 30.1 points per game.
- Rangi's average typing rate is 63 words per minute with an error rate of 2.3 errors per page.

4 b Integrate $\frac{d\theta}{dx}$ with respect to x . Use $\theta(0) = 0$ to show that $\theta(x) = x$ for all x .



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (Core) (3rd edition)

Third edition - 2012 initial print

The following errata also apply to books printed in 2012

page 66 **EXERCISE 2E 7 b**, should read:

7 Prove that:

- a** the sum of two even functions is an even function
- b** the difference between two odd functions is an **odd** function
- c** the product of two odd functions is an even function.

page 255 **EXAMPLE 14 Answer part c**, should read:

- c** The total number of combinations
 = the number with 3 men and 1 woman + the number with 2 men and 2 women
 + the number with 1 man and 3 women
 $= \binom{7}{3} \times \binom{6}{1} + \binom{7}{2} \times \binom{6}{2} + \binom{7}{1} \times \binom{6}{3}$
 $= 665$

page 267 **THE PROCESS OF INDUCTION**, at the bottom of the page should read:

The n th odd number is $(2n - 1)$, so we could also write the conjecture as:

$$1 + 3 + 5 + 7 + 9 + \dots + (2n - 1) = n^2 \quad \text{for all } n \in \mathbb{Z}^+$$

or $\sum_{i=1}^n (2i - 1) = n^2 \quad \text{for all } n \in \mathbb{Z}^+.$

page 275 **EXERCISE 9B.3 2 a**, should read:

2 Use the principle of mathematical induction to prove the following propositions:

- a** $3^n \geq 1 + 2n$ for all $n \in \mathbb{N}$
- b** $n! \geq 2^n$ for all $n \in \mathbb{Z}, n \geq 4$

page 493 **EXAMPLE 16**, should say in the hint:

a $2z = 2\sqrt{2} \operatorname{cis} \theta$
 $\therefore |2z| = 2\sqrt{2} \quad \text{and} \quad \arg(2z) = \theta$

The **range** of $\arg$ is
 $] -\pi, \pi]$.



page 619 **EXAMPLE 11**, should say in the first line of the answer:

Let x cm be the lengths of the sides of the cube, so the surface area $A = 6x^2 \text{ cm}^2$ and the volume $V = x^3 \text{ cm}^3$.

page 809 **REVIEW SET 25B 1**, should read:

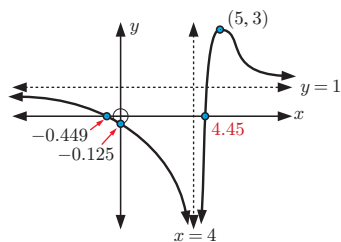
- 1** A **discrete** random variable X has probability distribution function $P(x) = k \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{3-x}$ where $x = 0, 1, 2, 3$ and k is a constant. Find:

page 850 **EXERCISE 27A 119 f**, should read:

- 119 f** The region bounded by $y = f(x)$, the **x -axis**, and the line $x = k$, $k > 0$, has area $\frac{1}{4}(e - 1)$ units². Find the value of k .

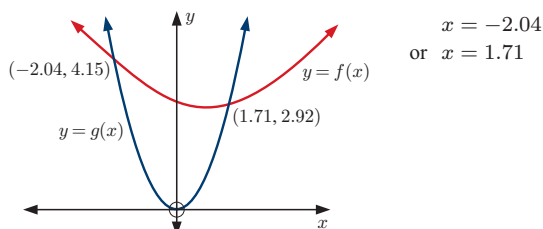
page 873 **ANSWERS EXERCISE 2K 1 c v**, should include all x -intercepts:

1 c v



page 873 **ANSWERS EXERCISE 2L 2**, graph for $y = f(x)$ should be:

2



page 873 **ANSWERS REVIEW SET 2A 5 b**, should have open intervals:

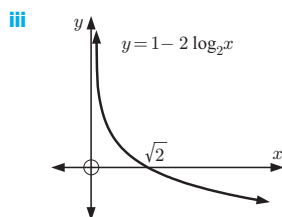
5 a $x = -1$ or 3 **b** $x \in]-\infty, -8[$ or $]2, \infty[$

page 877 **ANSWERS EXERCISE 3F 6 d iii**, should read:

6 d iii $y \approx 2.62$

page 880 **ANSWERS EXERCISE 4G 1 e**, should read:

- 1 e i** Domain is $\{x \mid x > 0\}$,
range is $\{y \mid y \in \mathbb{R}\}$
ii VA is $x = 0$,
 x -intercept $\sqrt{2}$,
no y -intercept
iv $x = 2$
v $f^{-1}(x) = 2^{\frac{1-x}{2}}$

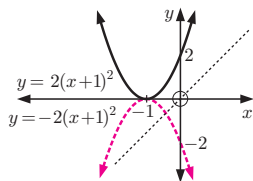


page 881 **ANSWERS EXERCISE 4H 5**, should read:

5 9 years

page 885 **ANSWERS EXERCISE 5D 1 f**, graphs should have x -intercept -1 :

1 f



page 885 **ANSWERS EXERCISE 5E 1 d**, should read:

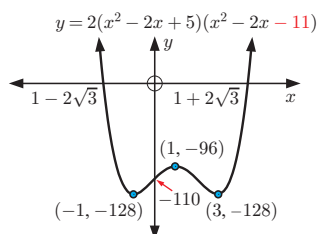
1 d A reflection in the x -axis, then a vertical stretch with scale factor 2.

page 890 **ANSWERS REVIEW SET 5B 11 c**, should read:

11 b $(1, 1)$ and $(2, -1)$ **c** VA is $x = \frac{3}{2}$, y -int is $\frac{1}{3}$

page 894 **ANSWERS REVIEW SET 6B 17 b**, graph should have function:

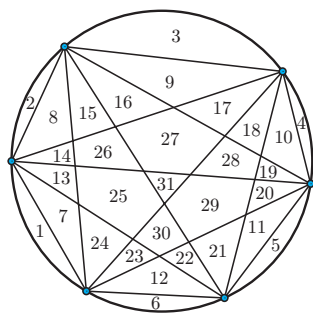
17 b



page 898 **ANSWERS EXERCISE 9A 5 b** and **5 c**, should read:

- 5 b** Conjecture: The number of regions for n points placed around a circle is given by $C_n = 2^{n-1}$, $n \in \mathbb{Z}^+$.

c $n = 6$



No. By the conjecture we expect $2^5 = 32$ regions, but there are only 31.

page 899 **ANSWERS EXERCISE 10D.2 2 b**, should read:

- 2 b** $\theta = 0$, π , or 2π

page 899 **ANSWERS EXERCISE 10E 3**, should read:

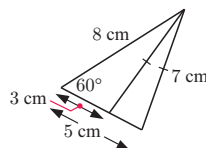
- 3** Hint: $\theta - \phi = -(\phi - \theta)$

page 900 **ANSWERS EXERCISE 11B 10 b**, should read:

- 10 a** $x = 2$ **b** $4\sqrt{6} \text{ cm}^2$ **11** $63^\circ, 117^\circ, 36^\circ, 144^\circ$

page 900 **ANSWERS REVIEW SET 11A 3 b** and **9 b ii**, should read:

- 3 a** $x = 3$ or 5 **b** Kady can draw 2 triangles, so she should ask for more information.



- 9 a** maximum value 16 when $x = 6$
b i $y = 12 - x$ **ii** $y^2 = x^2 - (16 \cos \theta)x + 64$

page 903 **ANSWERS EXERCISE 12E 2 c**, should read:

- 2 a** translation through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **b** reflection in x -axis
c horizontal stretch, factor 2 and vertical stretch with factor 2

page 906 **ANSWERS REVIEW SET 12C 7 a**, should read:

- 7 a** $y = \frac{3}{4} \sin(4x - 0.340) + \frac{1}{4}$ **b** $y = \tan(x - \frac{\pi}{2})$

page 906 **ANSWERS EXERCISE 13A.3 3 f**, should read:

- 3 e** $-\frac{\pi}{2} \leq 2(x - \frac{\pi}{4}) \leq \frac{7\pi}{2}$ **f** $-2\pi \leq -x \leq 0$

page 914 **ANSWERS EXERCISE 15A 2, 3, and 5**, should read:

- 2 a** 1.34 m s^{-1} in the direction 26.6° to the left of her intended line.
b i 30° to the right of Q **ii** 1.04 m s^{-1}
3 a 24.6 km h^{-1} **b** $\approx 9.93^\circ$ east of south
4 a 82.5 m **b** 23.3° to the left of straight across **c** 48.4 s
5 a The plane's speed in still air would be $\approx 437 \text{ km h}^{-1}$. The wind slows the plane down to 400 km h^{-1} .
b 4.64° north of due east

page 915 **ANSWERS EXERCISE 15D 1**, should read:

- 1** 75.7° **2** $\mathbf{b}_1 \cdot \mathbf{b}_2 = 0$ **3** 75.5°

page 917 **ANSWERS REVIEW SET 15B 6**, should read:

- 6** **a** $x_1(t) = 2 + t$, $y_1(t) = 4 - 3t$, $t \geq 0$
b $x_2(t) = 13 - t$, $y_2(t) = [3 - 2a] + at$, $t \geq 2$
c interception occurred at 2:22:30 pm
d bearing $\approx 12.7^\circ$ west of south, ≈ 4.54 units per minute

page 919 **ANSWERS EXERCISE 16B.1 4 b**, should read:

- 4** **a** 1 **b** $|r|$

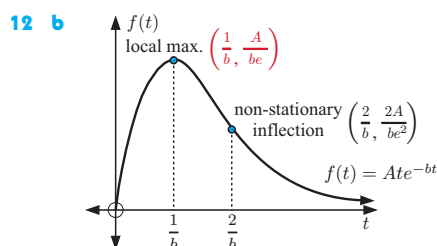
page 919 **ANSWERS EXERCISE 16C.1 3**, should read:

- 3** $k\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$ if $k > 0$, $-k\sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)$ if $k < 0$,
 not possible if $k = 0$

page 927 **ANSWERS EXERCISE 19B 7**, should read:

- 7** **a** decreasing for $-1 < x < 1$, never increasing
b increasing for $-1 < x < 1$, never decreasing

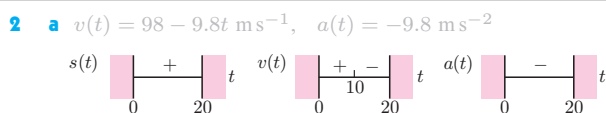
page 930 **ANSWERS EXERCISE 19D.1 12 b**, should read:



page 930 **ANSWERS REVIEW SET 19A 8 c**, should read:

- 8** **c** $f'(x) \leq 0$ for $x < 1$ and $1 < x \leq 2$
 and $f'(x) \geq 0$ for $x \geq 2$

page 932 **ANSWERS EXERCISE 20A.2 2 a**, should read:



page 933 **ANSWERS EXERCISE 20B 15**, should read:

- 15** $\frac{21}{\sqrt{2}} \text{ cm}^2$ per radian

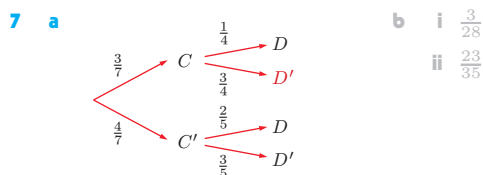
page 934 **ANSWERS EXERCISE 20C 30**, should read:

- 30** at grid reference (3.54, 8), $\approx 15.6 \text{ km}$

page 941 **ANSWERS EXERCISE 22C.2 8**, should read:

- 8** **a** $v(t) = \frac{1}{t+1} - 1 \text{ m s}^{-1}$ **b** $s(t) = \ln|t+1| - t$ metres
c $s(2) = \ln 3 - 2 \approx -0.901 \text{ m}$, $v(2) = -\frac{2}{3} \text{ m s}^{-1}$,
 $a(2) = -\frac{1}{9} \text{ m s}^{-2}$
 The object is approximately 0.901 m to the left of the origin,
 travelling left at $\frac{2}{3} \text{ m s}^{-1}$, with acceleration $-\frac{1}{9} \text{ m s}^{-2}$.

page 948 **ANSWERS REVIEW SET 24B 7 a**, should read:



page 949 ANSWERS EXERCISE 25A 3 a, should read:

3 a $X = 0, 1, 2, 3, \text{ or } 4$

page 951 ANSWERS EXERCISE 25F.3 3 a, should read:

3 a	x_i	0	1	2	3
	p_i	$(1 - p)^3$	$3p(1 - p)^2$	$3p^2(1 - p)$	p^3

page 951 ANSWERS EXERCISE 25G 1 b, should read:

1 a 1.5 b $p_x = \frac{(1.5)^x e^{-1.5}}{x!}$ for $x = 0, 1, 2, 3, 4, 5, 6, \dots$

x	0	1	2	3	4	5	6
f	12	18	12	6	3	0	1
$52p_x$	11.6	17.4	13.1	6.5	2.4	0.7	0.2

The fit is excellent.

page 952 ANSWERS REVIEW SET 25B 1, should read:

1 a $k = \frac{8}{5}$ b 0.975 c 2.55 d ≈ 0.740