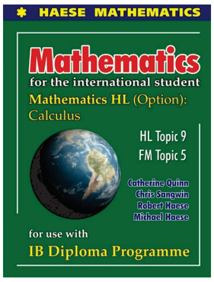


ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Calculus

First edition - 2017 fourth reprint

The following erratum was made on 28/Apr/2020

page 157 **Worked Solutions EXERCISE E** question **9**, last lines should read:

4 Hence, $\lim\limits_{x\to 0^+} x^{\sin x} = \lim\limits_{x\to 0^+} e^{\frac{\ln x}{[\sin x]^{-1}}}$

$= e^0$

$= 1$

The following erratum was made on 10/Jul/2019

page 192 **Worked Solutions EXERCISE L.4** question **4**, last lines should read:

4 By equating coefficients we get

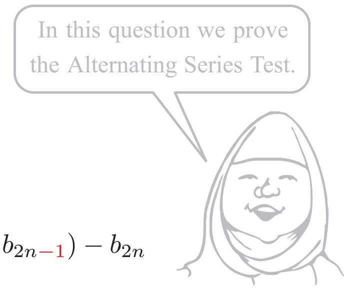
$a_0 = 0,\ \ a_1 = 1,\ \ a_2 = 0,\ \ a_3 = \frac{1}{3},\ \ a_4 = 0,\ \ a_5 = \frac{2}{15}$

$\therefore\ \ \tan x \approx x + \frac{1}{3}x^3 + \frac{2}{15}x^5$

The following erratum was made on 12/Apr/2018

page 83 **EXERCISE K.3** question **6 c**, should read:

6 Consider the alternating series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}b_n$ where $0 \leqslant b_{n+1} \leqslant b_n$ for all $n \in \mathbb{Z}^+$ and $\lim\limits_{n\to\infty} b_n = 0$.



a Explain why $S_2 = b_1 - b_2 \geqslant 0$.

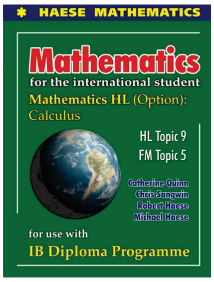
b Show that $S_4 \geqslant S_2$. Hence prove that in general, $S_{2n} \geqslant S_{2n-2}$ and $0 \leqslant S_2 \leqslant S_4 \leqslant .... \leqslant S_{2n} \leqslant ....$.

c Show that $S_{2n} = b_1 - (b_2 - b_3) - (b_4 - b_5)\ .... - (b_{2n-2} - b_{2n-1}) - b_{2n}$ and $S_{2n} \leqslant b_1$.

d Hence prove that S_{2n} is convergent. Let $\lim\limits_{n\to\infty} S_{2n} = S$.

e Show that $S_{2n+1} = S_{2n} + b_{2n+1}$.

f Show that if $\lim\limits_{n\to\infty} b_n = 0$ then $\lim\limits_{n\to\infty} S_{2n+1} = S$ and hence $\lim\limits_{n\to\infty} S_n = S$.



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Calculus

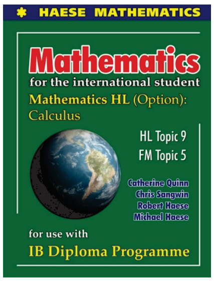
First edition - 2015 third reprint

The following erratum was made on 27/Feb/2017

page 55 **SECTION I DECREASING AND INCREASING FUNCTIONS,** Last line before Example 20 should specify:

Similarly, for any negative continuous function which is **increasing on** $[a, \infty[$,

$$L = \sum_{i=a}^{\infty} f(i) \leqslant \int_a^{\infty} f(x)\, dx \leqslant \sum_{i=a}^{\infty} f(i+1) = U.$$



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Calculus

First edition - 2015 second reprint

The following errata were made on 14/Nov/2016

page 106 **SECTION M Example 47,** solutions to parts **c** and **d** should read:

c $y = ce^{3x} - 1$ is a general solution to the differential equation.

The particular solution passes through $(0, 2)$, so $2 = ce^{3\times 0} - 1$

$\therefore\ c = 3$

$\therefore$ the particular solution is $y = 3e^{3x} - 1$

d $\dfrac{dy}{dx} = 3 + 3y$

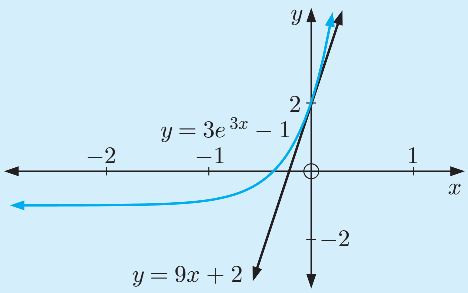
$\therefore$ at the point $(0, 2)$, $\dfrac{dy}{dx} = 3 + 3\times 2 = 9$

$\therefore$ the gradient of the tangent to the particular solution $y = 3e^{3x} - 1$ at $(0, 2)$, is 9.

$\therefore$ the equation of the tangent is

$\dfrac{y-2}{x-0} = 9$

$\therefore\ y = 9x + 2$



page 114 **SECTION N Example 53,** first line of solution should read:

Since OP = PQ, triangle OPQ is isosceles. Hence [PA] is the perpendicular bisector of [OQ].

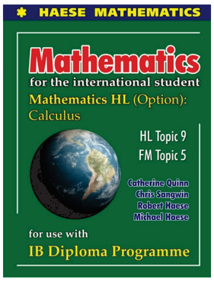
The following errata were made on or before 09/Dec/2015

page 171 **Worked Solutions EXERCISE K.1,** Question **1 d** had an invalid first step, replace whole solution with:

1 d Suppose $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ converges

$\therefore\ \displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right) + \sum_{n=1}^{\infty}\frac{1}{n^2} = \sum_{n=1}^{\infty}\frac{1}{n}$ converges

This is false $\therefore\ \displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ diverges.



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Calculus

First edition - 2014 initial print

The following errata were made on or before 18/Aug/2015

page 19 **EXERCISE B.3,** Question **3 c** was bad. Replace with **3 d** and add Question 4:

3 Use the Squeeze Theorem to prove that $\lim\limits_{x\to 0} g(x) = 0$ for:

a $g(x) = x^2\cos\left(\frac{1}{x^2}\right)$ **b** $g(x) = x\sin\left(\frac{1}{x}\right)$ **c** $g(x) = \dfrac{|x|}{1+x^4}$

4 **a** Use the Squeeze Theorem to prove that $\lim\limits_{x\to 0^+} e^{\left(-\frac{1}{x}\right)}\sin x = 0$.

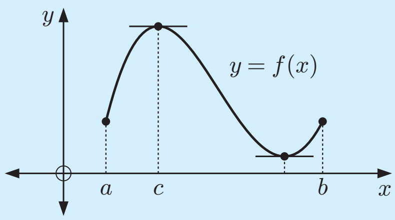
b Explain why $\lim\limits_{x\to 0} e^{\left(-\frac{1}{x}\right)}\sin x$ does not exist.

page 34 **SECTION F ROLLE'S THEOREM,** Rolle's Theorem doesn't require $f(a) = f(b) = 0$:

ROLLE'S THEOREM

Suppose function $f : D \to \mathbb{R}$ is continuous on the closed interval $[a, b]$, and differentiable on the open interval $]\,a, b\,[$.

If $f(a) = f(b)$, then there exists a value $c \in\,]\,a, b\,[$ such that $f'(c) = 0$.



Proof of Rolle's theorem:

Since f is continuous on $[a, b]$, it attains both a maximum and minimum value on $[a, b]$.
If f takes values greater than $f(a)$ on $[a, b]$, let $f(c)$ be the maximum of these.

Now $f(b) = f(a)$ and $f(c) > f(a)$, so $c \in\,]\,a, b\,[$.

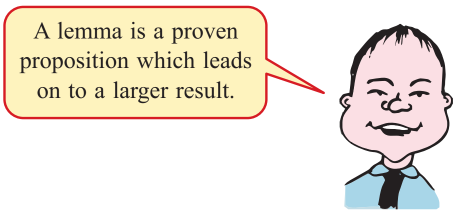
Since f is differentiable at c, f must have a local maximum at $x = c$. $\therefore$ $f'(c) = 0$.

Similarly, if f takes values less than $f(a)$ on $[a, b]$, let the minimum of these be $f(c)$.
It follows that f has a local minimum at $x = c$, and therefore $f'(c) = 0$.

Finally, if $f(x) = f(a)$ for all $x \in [a, b]$ then clearly $f'(c) = 0$ for all $c \in\,]\,a, b\,[$.

By taking $f(a) = f(b) = 0$, Rolle's theorem guarantees that between any two zeros of a differentiable function f there is *at least one* point at which the tangent line to the graph $y = f(x)$ is horizontal.

Rolle's theorem is a *lemma* used to prove the **Mean Value Theorem**.



page 49 **EXERCISE H** question **8,** should ask for definite intervals within the domain of $F(x)$:

8 Let $F(x) = \displaystyle\int_1^x \cos(e^{t^2})\,dt$, $x > 1$. Find exactly:

a $F'(x)$ **b** $F'(2)$ **c** $F'\left(\sqrt{\ln\pi}\right)$ **d** $F''(x)$ **e** $F''(2)$

page 61 **SECTION J SEQUENCES,** Remove Comparison Test for Sequences:

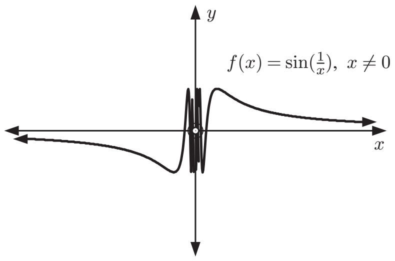
~~COMPARISON TEST FOR SEQUENCES~~

~~Consider sequences $\{a_n\}$, $\{b_n\}$ such that $0 \leqslant a_n \leqslant b_n$.~~
~~If $\{a_n\}$ diverges, then $\{b_n\}$ diverges.~~
~~If $\{b_n\}$ converges, then $\{a_n\}$ converges.~~

note: Counter example $\{a_n\} = 1,2,1,2,1,2, \ldots$ and $\{b_n\} = 3, 3, 3, 3, \ldots$

page 147 **Worked Solutions EXERCISE B.1** question **7 c,** the limits do not exist:

7 c



$f(x)$ does not approach any value as $x \to 0$ from above or below.

i $\lim\limits_{x\to 0^-} f(x)$ DNE **ii** $\lim\limits_{x\to 0^+} f(x)$ DNE

iii $\lim\limits_{x\to 0} f(x)$ DNE

page 149 **Worked Solutions EXERCISE B.3** question **3 d** becomes **3 c,** and question **4** is added:

4 a As $-1 \leqslant \sin x \leqslant 1$ and $e^{\frac{1}{x}} > 0$ for all $x \in \mathbb{R}$,

$$-\frac{1}{e^{\frac{1}{x}}} \leqslant \frac{\sin x}{e^{\frac{1}{x}}} \leqslant \frac{1}{e^{\frac{1}{x}}}$$

But as $x \to 0^+$, $\frac{1}{x} \to \infty$ and $e^{\frac{1}{x}} \to \infty$

$$\therefore \lim_{x\to 0^+}\left(-\frac{1}{e^{\frac{1}{x}}}\right) = \lim_{x\to 0^+}\left(\frac{1}{e^{\frac{1}{x}}}\right) = 0$$

$$\therefore \lim_{x\to 0^+} e^{\left(-\frac{1}{x}\right)} \sin x = 0 \quad \{\text{Squeeze Theorem}\}$$

b As $-1 \leqslant \sin x \leqslant 1$ and $e^{\frac{1}{x}} > 0$ for all $x \in \mathbb{R}$,

$$-\frac{1}{e^{\frac{1}{x}}} \leqslant \frac{\sin x}{e^{\frac{1}{x}}} \leqslant \frac{1}{e^{\frac{1}{x}}}$$

Now $\lim\limits_{x\to 0^+} e^{-\frac{1}{x}} = 0$ but $\lim\limits_{x\to 0^-} e^{-\frac{1}{x}}$ is undefined.

$\therefore \lim\limits_{x\to 0} e^{-\frac{1}{x}}$ is undefined.

$\therefore \lim\limits_{x\to 0} e^{-\frac{1}{x}} \sin x$ is undefined.

page 161 **Worked Solutions EXERCISE H** question **8,** answers need to change according to question alteration:

8 $F(x) = \displaystyle\int_1^x \cos(e^{t^2})\,dt, \quad x > 1$

a $F'(x) = \cos(e^{x^2}), \quad x > 1 \qquad \{\text{FTOC}\}$

b $F'(2) = \cos(e^4)$

c $F'\left(\sqrt{\ln \pi}\right) = \cos\left(e^{\ln \pi}\right)$
$= \cos \pi$
$= -1$

d $F''(x) = -\sin(e^{x^2})\, e^{x^2}\, 2x$
$= -2x\, e^{x^2} \sin(e^{x^2})$

e $F''(2) = -4e^4 \sin(e^4)$

page 179 **Worked Solutions EXERCISE K.3** questions **7 e** and **f**, the series are conditionally convergent:

7 **e** Consider $f(x) = \dfrac{\ln x}{x}$

Now $f'(x) = \dfrac{\frac{1}{x}x - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2}$

$\therefore$ $f'(x) < 0$ for all x such that $\ln x > 1$, that is, $x > e$

Thus $\left\{\dfrac{\ln n}{n}\right\}$ is decreasing for all $n \geqslant 3$, $n \in \mathbb{Z}^+$ .... (1)

By l'Hôpital's Rule, $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{x}}{1} = 0$

$\therefore$ $\lim\limits_{n\to\infty} \dfrac{\ln n}{n} = 0$, $n \in \mathbb{Z}^+$ .... (2)

$\therefore$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln n}{n}$ is convergent {Alternating Series Test}

But $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ is divergent {Integral test}

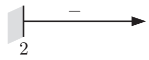
$\therefore$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln n}{n}$ is conditionally convergent.

f Let $b_n = \dfrac{1}{n\ln n}$, then $\lim\limits_{n\to\infty} b_n = 0$

Now consider $f(x) = \dfrac{1}{x\ln x} = [x\ln x]^{-1}$, $x \geqslant 2$

$\therefore$ $f'(x) = -[x\ln x]^{-2}\left(1\ln x + x\left(\dfrac{1}{x}\right)\right)$

$= \dfrac{-(\ln x + 1)}{(x\ln x)^2}$



So, $f'(x) < 0$ for all $x \geqslant 2$

$\Rightarrow$ $b_n = \dfrac{1}{n\ln n}$ is decreasing for all $n \geqslant 2$, $n \in \mathbb{Z}^+$

Thus $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ is convergent.

But $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ is divergent {Integral test}

$\therefore$ $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ is conditionally convergent.

page 184 **Worked Solutions EXERCISE K.4** question **8**, the last line of the solution should have correct interval:

8 ....

$\therefore$ $\displaystyle\sum_{n=0}^{\infty} \frac{nx^{n-1}}{n^2 3^n}$ converges for all $x \in [-3, 3[$, and has radius of convergence 3.

page 187 **Worked Solutions EXERCISE L.2** question **2 a**, should read:

2 **a**

$f(x) = e^{-x}$, $\therefore$ $f(0) = 1$

$f'(x) = -e^{-x}$, $\therefore$ $f'(0) = -1$

$f''(x) = e^{-x}$, $\therefore$ $f''(0) = 1$

$f'''(x) = -e^{-x}$, $\therefore$ $f'''(0) = -1$

By Maclaurin's theorem,

$$f(x) = e^{-x} = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^{(n)}(0)x^n}{n!} + R_n(x:0)$$

....