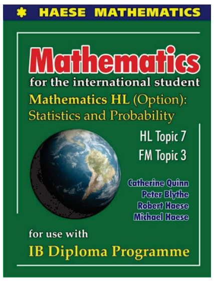


ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Statistics and Probability

First edition - 2017 third reprint

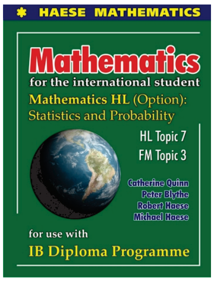
The following erratum was made on 28/Apr/2020

page 160 **Worked Solutions EXERCISE A 4 a ii,** should read:

4 a ii

y	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

y^2	1	4	9	16	25	36
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Statistics and Probability

First edition - 2015 second reprint

The following erratum was made on 18/Sep/2017

page 167 **Worked Solutions EXERCISE B.4 8 e**, should read:

8 **e** As $\mu \approx \sigma^2$, $n \geqslant 50$ and $p \leqslant 0.1$ a Poisson distribution could be used to approximate the binomial distribution where $X \sim \text{Po}(9.56)$.

i $\text{P}(X \leqslant 4) \approx 0.0388$

ii $\text{P}(X \geqslant 6) = 1 - \text{P}(X \leqslant 5)$

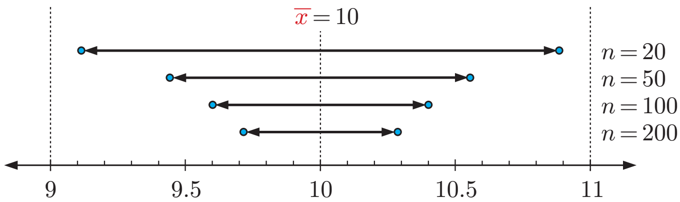
≈ 0.914

The following errata were made on 14/Nov/2016

page 92 **OTHER CONFIDENCE INTERVALS FOR u**, diagram at bottom of page:

For various values of n we have:

n	*Confidence interval*
20	$9.123 \leqslant \mu \leqslant 10.877$
50	$9.446 \leqslant \mu \leqslant 10.554$
100	$9.608 \leqslant \mu \leqslant 10.392$
200	$9.723 \leqslant \mu \leqslant 10.277$



page 203 **Worked Solutions REVIEW SET D 6 c ii**, last line of solution should read:

6 **c** **ii** $\therefore$ we have a 93% confidence level.

The following errata were made on 24/May/2016

page 97 **EXERCISE G.1 Question 9 b**, should read:

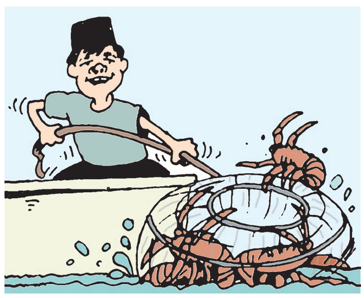
9 A sample of 60 yabbies was taken from a dam. The sample mean weight of the yabbies was 84.6 grams, and the sample standard deviation was 16.8 grams.

a For this yabbie population, find:

i the 95% confidence interval for the population mean

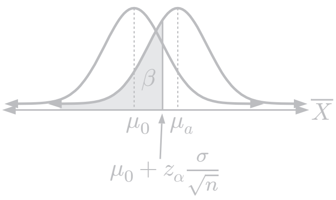
ii the 99% confidence interval for the population mean.

b Suppose the population standard deviation $\sigma = 16.94$ g. What sample size is needed to be 95% confident that the sample mean differs from the population mean by less than 5 g?



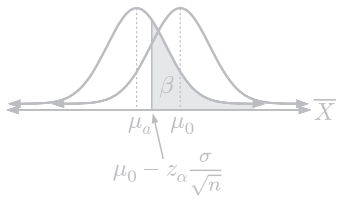
page 115 **SECTION H THE PROBABILITY OF ERROR,** should not have the second equation in the **Two-tailed test**:

One-tailed (right) test	One-tailed (left) test	Two-tailed test
test distribution, true distribution	true distribution, test distribution	test distribution, true distribution



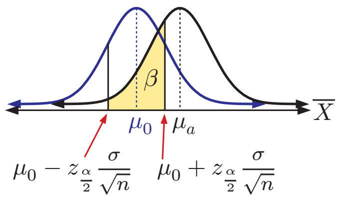
$\beta = \mathrm{P}(\overline{X} \leqslant \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}})$

where $\overline{X} \sim \mathrm{N}(\mu_a, \frac{\sigma^2}{n})$



$\beta = \mathrm{P}(\overline{X} \geqslant \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}})$

where $\overline{X} \sim \mathrm{N}(\mu_a, \frac{\sigma^2}{n})$



$\beta = \mathrm{P}(\mu_0 - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leqslant \overline{X} \leqslant \mu_0 + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$

where $\overline{X} \sim \mathrm{N}(\mu_a, \frac{\sigma^2}{n})$

page 182 **Worked Solutions EXERCISE G.1 9 b**, should read:

9 **b** The 95% confidence interval for μ is

$$\therefore\ 84.6 - 1.96\frac{\sigma}{\sqrt{n}} \leqslant \mu \leqslant 84.6 + 1.96\frac{\sigma}{\sqrt{n}}$$

$$\therefore\ -1.96\frac{\sigma}{\sqrt{n}} \leqslant \mu - 84.6 \leqslant 1.96\frac{\sigma}{\sqrt{n}}$$

Thus $1.96 \times \frac{16.94}{\sqrt{n}} < 5$

$$\therefore\ \sqrt{n} > \frac{1.96 \times 16.94}{5}$$

$$\therefore\ n > 44.10$$

$\therefore$ a sample of 45 or more is needed.

page 188 **Worked Solutions EXERCISE H.6 1 b**, should read:

1 **a** H_0: The die is fair for rolling a '4', so $p = \frac{1}{4}$

H_1: The die is unfair, so $p \neq \frac{1}{4}$

b **i** A Type I error is rejecting H_0 when it is true. This means deciding it is biased when it is in fact fair.

ii P(Type I error)

$= \mathrm{P}(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= \mathrm{P}(X \leqslant 61 \text{ or } X \geqslant 89 \mid p = \frac{1}{4})$

where $X \sim \mathrm{B}(300, \frac{1}{4})$

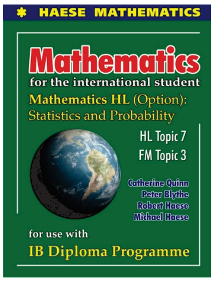
$= 1 - \mathrm{P}(62 \leqslant X \leqslant 88)$ with $X \sim \mathrm{B}(300, \frac{1}{4})$

$= 1 - [\mathrm{P}(X \leqslant 88) - \mathrm{P}(X \leqslant 61)]$

$\approx 1 - 0.962\,26 + 0.033\,785$

≈ 0.0715

iii The test is about the 7.15% level.



ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

MATHEMATICS HL (Option): Statistics and Probability

First edition - 2013 initial print

The following errata were made on or before 13/Jul/2015

page 130 **EXERCISE I.1 Questions 6 b** and **c**, should read:

6 **b** Let $u = 2x + 1$ and $v = 3y - 1$.

i List the data (u, v) in a table. **ii** Calculate r for this data.

c Let $u = -2x + 1$ and $v = 3y - 1$.

i List the data (u, v) in a table. **ii** Calculate r for this data.

page 151 **REVIEW SET C Question 1 d**, should read:

1 **d** Find $\mathrm{P}(Y \geqslant 8b)$.

page 154 **REVIEW SET D Question 1 d**, should say that X is normally distributed:

7 In order to estimate the copper content of a potential mine, drill core samples are used. All of the drill core is crushed and well mixed before samples are removed.

Suppose X is the copper content in grams per kilogram of core, and that X has mean $\mu = 11.4$ and $\sigma = 9.21$.

page 167 **Worked Solutions EXERCISE B.4 5**, should read:

5 **A** $X \sim \mathrm{Po}(6)$. $\mathrm{P}(X = 3) \approx 0.0892$

B $X \sim \mathrm{Po}(1)$. $\mathrm{P}(X = 1) \approx 0.3679$

C $X \sim \mathrm{Po}(24)$. $\mathrm{P}(X \leqslant 16) \approx 0.0563$

As **B** has the highest probability it is the most likely to occur.

page 168 **Worked Solutions EXERCISE C 4 a**, should read:

4 **a** We require $\int_0^k (6 - 18x)\,dx = 1$

$$\therefore \left[6x - 9x^2\right]_0^k = 1$$
$$\therefore 6k - 9k^2 = 1$$
$$\therefore 9k^2 - 6k + 1 = 0$$
$$\therefore (3k - 1)^2 = 0$$
$$\therefore k = \tfrac{1}{3}$$

page 169 **Worked Solutions EXERCISE C 6**, should read:

6 **c** $X \sim \mathrm{N}(2.5, 2.5)$

$\therefore \mathrm{P}(X > 2) = \mathrm{P}(X^* \geqslant 2.5)$
≈ 0.500

$Y \sim \mathrm{N}(130, 130)$

$\therefore \mathrm{P}(Y > 104) = \mathrm{P}(Y^* \geqslant 104.5)$
≈ 0.987

The approximation for X is poor, but that for Y is very good. This is probably due to the fact that $2.5 = \lambda$ is not large enough.

page 172 **Worked Solutions EXERCISE D.3 2 b**, should read:

2 **b** $G'(t) = \frac{1}{6} + \frac{2}{3}t + \frac{3}{2}t^2$ and

$G''(t) = \frac{2}{3} + 3t$

Thus $\mathrm{E}(X) = G'(1) = \frac{1}{6} + \frac{2}{3} + \frac{3}{2}$

$\therefore \mathrm{E}(X) = 2\frac{1}{3}$

and $\mathrm{Var}(X) = G''(1) + G'(1) - [G'(1)]^2$

$= 3\frac{2}{3} + 2\frac{1}{3} - (2\frac{1}{3})^2$

$= \frac{5}{9}$

page 177 **Worked Solutions EXERCISE E.2 13**, should read:

13 $\mu_X = 74$ and $\sigma_X = 6$

$$\overline{X} \sim \mathrm{N}\left(74, \frac{6^2}{n}\right)$$

$$\mathrm{P}(\overline{X} < 70.4) = 0.001\,35$$

$$\therefore \mathrm{P}\left(\frac{\overline{X}-74}{\frac{6}{\sqrt{n}}} < \frac{70.4-74}{\frac{6}{\sqrt{n}}}\right) = 0.001\,35$$

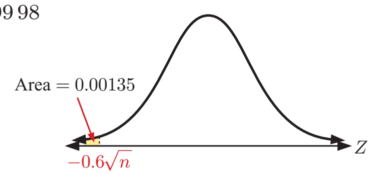
$$\therefore \mathrm{P}\left(Z < \frac{-3.6\sqrt{n}}{6}\right) = 0.001\,35$$

$$\therefore \mathrm{P}\left(Z < -0.6\sqrt{n}\right) = 0.001\,35$$

$\therefore -0.6\sqrt{n} \approx -2.999\,98$

$\therefore \sqrt{n} \approx 5$

$\therefore n \approx 25$



page 178 **Worked Solutions EXERCISE E.2 17 c** and **18 a iv**, should read:

17 **c** Let the contents of the three small cartons be $\overline{S_1}$, $\overline{S_2}$, and $\overline{S_3}$ and consider

$V = \overline{E} - (\overline{S_1} + \overline{S_2} + \overline{S_3})$

$\mathrm{E}(V) = \mathrm{E}(\overline{E}) - \mathrm{E}(\overline{S_1}) - \mathrm{E}(\overline{S_2}) - \mathrm{E}(\overline{S_3})$

$= 950 - 315 - 315 - 315$

$= 5$

$\mathrm{Var}(V) = \mathrm{Var}(\overline{E}) + \mathrm{Var}(\overline{S_1}) + \mathrm{Var}(\overline{S_2}) + \mathrm{Var}(\overline{S_3})$

$= \frac{25}{10} + 3(\frac{4}{15})$

$= 3.3$

$\therefore V \sim \mathrm{N}(5, 3.3)$

$\mathrm{P}(\overline{E} < \overline{S_1} + \overline{S_2} + \overline{S_3}) = \mathrm{P}(\overline{E} - (\overline{S_1} + \overline{S_2} + \overline{S_3}) < 0)$

$= \mathrm{P}(V < 0)$

$\approx 0.002\,96$

18 **a** **i** Let $X =$ the number of people cured then $X \sim \mathrm{B}(100, \frac{3}{4})$.

ii $\mu_X = np$
$= 100 \times \frac{3}{4}$
$= 75$

$\sigma_X = \sqrt{np(1-p)}$
$= \sqrt{100 \times \frac{3}{4} \times \frac{1}{4}}$
≈ 4.3301

iii $\mathrm{P}(X \leqslant 68) \approx 0.0693$

iv $\overline{X} \sim \mathrm{N}(0.75, \frac{\frac{3}{4} \times \frac{1}{4}}{100})$ {CLT}

$\therefore \mathrm{P}(\overline{X} \leqslant 0.68) \approx 0.0530$

page 178 **Worked Solutions EXERCISE E.3 1 a, 4 b and d, should read:**

1 **a** Claim is: $p = 0.04$ and $n = 1000$
As $np = 40$ and $n(1-p) = 960$ are both $\geqslant 5$ we can assume that $\widehat{p} \sim \mathrm{N}\left(0.04, \dfrac{0.04 \times 0.96}{1000}\right)$
$\mathrm{P}(\widehat{p} \geqslant 0.07) \approx 6.46 \times 10^{-7}$

4 **b** For $\widehat{p}$ to be approximated by the normal distribution we require that

$$\begin{aligned} np &\geqslant 5 \quad \text{and} \quad n(1-p) \geqslant 5 \\ \therefore\ 0.85n &\geqslant 5 \quad \text{and} \quad 0.15n \geqslant 5 \\ \therefore\ n &\geqslant 5.88 \quad \text{and} \quad n \geqslant 33.33 \\ \therefore\ n &\geqslant 34 \end{aligned}$$

d **i** $n = 500$, $\widehat{p} = \frac{350}{500} = 0.7$ and $p = 0.85$
$np = 425$ and $n(1-p) = 75$ are both $\geqslant 5$

page 179 **Worked Solutions EXERCISE E.3 6 a, should read:**

6 **a** $n = 250$ and their claim is $p = 0.9$
$np = 250 \times 0.9 = 225$ and $n(1-p) = 25$ and these are both $\geqslant 5$

page 182 **Worked Solutions EXERCISE G.1 9 and 10 c, should read:**

9 $n = 60$, $\overline{x} = 84.6$, $s_n = 16.8$

$$s_{n-1} = \sqrt{\frac{n}{n-1}}\, s_n = \sqrt{\tfrac{60}{59}} \times 16.8$$

$\therefore\ s_{n-1} \approx 16.94$ {unbiased estimate of σ}

As we had to estimate σ from s_n, the t-distribution applies.

a **i** 95% confidence interval is $80.3 < \mu < 89.0$
ii 99% confidence interval is $78.8 < \mu < 90.4$

b The 95% confidence interval for μ is

$$\therefore\ 84.6 - \frac{s_{n-1}}{\sqrt{n}}\, t_{0.025} \leqslant \mu \leqslant 84.6 + \frac{s_{n-1}}{\sqrt{n}}\, t_{0.025}$$

$$\therefore\ -2.001\,\frac{s_{n-1}}{\sqrt{n}} \leqslant \mu - 84.6 \leqslant 2.001\,\frac{s_{n-1}}{\sqrt{n}}$$

Thus $2.001 \times \dfrac{16.94}{\sqrt{n}} < 5$

$$\therefore\ \sqrt{n} > \frac{2.001 \times 16.94}{5}$$

$\therefore\ n > 45.97$
$\therefore$ a sample of 46 or more is needed.

10 $\sum x = 112.5$ and $\sum x^2 = 1325.31$

a $\overline{x} = \dfrac{\sum x}{n} = \frac{112.5}{10} = 11.25$

b $s_n^{\,2} = \dfrac{\sum x^2}{n} - \overline{x}^2 = \frac{1325.31}{10} - 11.25^2 = 5.9685$

$$s_{n-1} = \sqrt{\frac{n}{n-1}}\, s_n$$

gives $s_{n-1} \approx 2.575$ and s_{n-1} is an unbiased estimate of σ.

c As s_{n-1} is used as an unbiased estimate of σ^2 the t-distribution applies.
From technology, $9.76 \leqslant \mu \leqslant 12.7$

page 183 **Worked Solutions EXERCISE G.1 12 b** should read:

12 **b** As s_{n-1} is used to estimate σ, we use the t-distribution.
The 99% confidence interval for μ is

$$\overline{x} - \frac{s_{n-1}}{\sqrt{n}} t_{0.005} \leqslant \mu \leqslant \overline{x} + \frac{s_{n-1}}{\sqrt{n}} t_{0.005}$$

$$\therefore \quad -2.685 \frac{s_{n-1}}{\sqrt{n}} \leqslant \mu - \overline{x} \leqslant 2.685 \frac{s_{n-1}}{\sqrt{n}}$$

$\therefore$ we require $\dfrac{2.685 \times 4.7497}{\sqrt{n}} < 1.8$

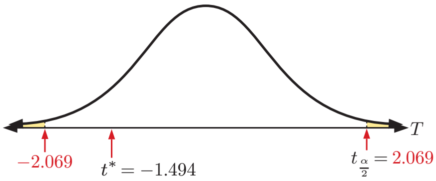
$$\therefore \quad \sqrt{n} > \frac{2.685 \times 4.7497}{1.8}$$

$$\therefore \quad n > 50.2$$

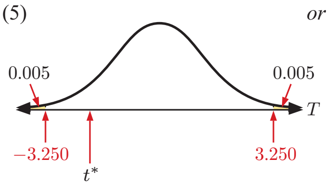
$\therefore$ n should be at least 51.

page 185 **Worked Solutions EXERCISE H.3** Questions **2 b i** and **4** (5) should read:

2 **b** **i**



4 (5)



or

p-value
$= 2 \times \mathrm{P}(T \geqslant |t^*|)$
$= 2 \times \mathrm{P}(T \geqslant 2.5)$
$\approx 0.033\,86$

page 186 **Worked Solutions EXERCISE H.4** Question **1** (4) should read:

1 (4) We reject H_0 if:
t^* lies in the critical region *or* the p-value is < 0.05

page 187 **Worked Solutions EXERCISE H.4** Questions **2** (4) and **3** (4) should read:

2 (4) We reject H_0 if:
t^* lies in the critical region *or* the p-value is < 0.05

3 (4) We reject H_0 if:
t^* lies in the critical region *or* the p-value < 0.05

page 190 **Worked Solutions EXERCISE H.6** Question **6 a ii** should read:

6 **a** **ii** $\mathrm{P}(5 \text{ or } 6 \text{ sixes} \mid X \sim \mathrm{B}(6, \frac{1}{6}))$
$= \mathrm{P}(X \geqslant 5 \mid X \sim \mathrm{B}(6, \frac{1}{6}))$
$= 1 - \mathrm{P}(X \leqslant 4 \mid X \sim \mathrm{B}(6, \frac{1}{6}))$
$\approx 0.000\,664$

page 191 **Worked Solutions EXERCISE I.1 2 b** should read:

2 **b** $\sum x_i y_i = 16 + 20 + 24 + 28 + 24 + 30 + 36 + 32 + 40 + 40$
$= 290$

$\sum {x_i}^2 = 4 + 4 + 4 + 4 + 9 + 9 + 9 + 16 + 16 + 25$
$= 4 \times 4 + 3 \times 9 + 2 \times 16 + 25$
$= 100$

$\sum {y_i}^2 = 8^2 + 10^2 + 12^2 + 14^2 + 8^2 + 10^2 + 12^2 + 8^2 + 10^2 + 8^2$
$= 4 \times 8^2 + 3 \times 10^2 + 2 \times 12^2 + 14^2$
$= 1040$

$$\therefore \quad r = \frac{290 - 10 \times 3 \times 10}{\sqrt{(100 - 10 \times 3^2)(1040 - 10 \times 10^2)}}$$
$$= \frac{-10}{\sqrt{10 \times 40}}$$
$$= \frac{-10}{20}$$
$$= -0.5$$

Technology gives the values for $\overline{x}$, $\overline{y}$, n, $\sum {x_i}^2$, $\sum {y_i}^2$, $\sum x_i y_i$, and r.

page 193 **Worked Solutions EXERCISE I.3** Questions **1**, **2** and **3** should read:

1 .....
(4) At a 5% level, we do not accept H_0. We accept that the data is correlated.
At a 1% level, we accept H_0 that the data is not correlated.

2 .....
(4) At both the 5% level and 1% level we do not accept H_0. We accept that the data is correlated.

3 .....
(4) At a 5% level we do not accept H_0. We accept that the data is correlated.
At a 1% level we accept H_0 that the data is not correlated.

page 195 **Worked Solutions REVIEW SET A 4 b**, should read:

4 **b** $X \sim \mathrm{B}(1050, 0.75)$
Now $np > 5$ and $n(1-p) > 5$
$\therefore$ we can approximate X by a normal variate with
$\mu = np = 787.5$ and $\sigma = \sqrt{np(1-p)} = 14.03$

page 200 **Worked Solutions REVIEW SET C 1 d**, should read:

1 **d** $\mathrm{P}(Y \geqslant 8b) = \mathrm{P}(Y \geqslant 1.25)$
≈ 0.106

page 203 **Worked Solutions REVIEW SET D 3 b ii**, should read:

3 **b** **ii** As $n \geqslant 30$, we can use the Central Limit Theorem.
$\overline{X} \sim \mathrm{N}(8, \frac{6}{32})$
$\therefore \quad \mathrm{P}(7.9 \leqslant \overline{X} \leqslant 8.1) \approx 0.183$